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Cycle saturation in random graphs

Given two graphs G and F, an inclusion-maximum F-free spanning subgraph $H \subset G$ is called an *F-saturated subgraph of G*. The minimum number of edges in an F-saturated subgraph of G is called *F-saturation number of G* and denoted by $\mathrm{sat}(G, F)$. The stability of $\mathrm{sat}(K_n, F)$ was studied by Korandi and Sudakov in 2017 for complete graphs and by Mohammadian and Tayfeh-Rezaie in 2018 for star graphs.

In 1972, Ollmann proved that $\mathrm{sat}(K_n, C_4) = \lfloor \frac{3n-5}{2} \rfloor$ for all $n \geq 5$. In 2009, Chen proved that $\mathrm{sat}(K_n, C_5) = \lceil \frac{10}{7}(n-1) \rceil$ for $n \geq 21$. For all other ℓ, the exact value of $\mathrm{sat}(K_n, C_\ell)$ is not known. However, several non-trivial bounds are obtained. In particular, $\mathrm{sat}(K_n, C_\ell) > (1+\varepsilon(\ell))n$ for some $\varepsilon(\ell) > 0$. We have proved that $\mathrm{sat}(K_n, C_\ell)$ is not asymptotically stable for all $\ell \geq 5$: with high probability $\mathrm{sat}(G(n,p), C_\ell) = n(1+o(1))$. We have also proved that there exists γ such that with high probability $\mathrm{sat}(G(n,p), C_4) \leq \gamma n$.

This is joint work with Yury Demidovich.