
New Trends in Analytic Combinatorics
(Org: **Stephen Melczer** (University of Waterloo))

STEPHEN GILLEN, University of Pennsylvania

Gillis-Reznick-Zeilberger's power series and the mysterious factor of 3

Analytic combinatorics in several variables (ACSV) is the study of approximating series coefficients of generating functions asymptotically using methods from topology and complex analysis. We examine two examples of generating functions with "lacunas", where the minimal singularity does not contribute asymptotically and lower critical points dominate. In one example, the coefficient asymptotics are exactly what would be expected from the smooth-point formula at the lower critical points, but in the other, they appear to be off by a factor of three. We aim to determine the source of the enigmatic factor of three.

MARCUS MICHELEN, University of Illinois at Chicago

Maximum entropy and integer partitions

We derive asymptotic formulas for the number of integer partitions with given sums of j th powers of the parts for j belonging to a finite, non-empty set $J \subset \mathbb{N}$. The method we use is based on the "principle of maximum entropy" of Jaynes. This principle leads to an intuitive variational formula for the asymptotics of the logarithm of the number of constrained partitions as the solution to a convex optimization problem over real-valued functions. Based on joint work with Gweneth McKinley and Will Perkins.

GRETA PANOVA, University of Southern California

Unimodality and Kronecker asymptotics via random variables

Cayley conjectured, and Sylvester proved some 25 years later, that the number of integer partitions of n inside a rectangle is a unimodal sequence of n . All subsequent proofs relied on representation theory, linear algebra and combinatorics without effective tight bounds for the size of these numbers. We derive tight asymptotics for these numbers and their consecutive differences using tilted geometric random variables. As a corollary, we have exact asymptotics for a family of Kronecker coefficients of the Symmetric group.

VERONIKA PILLWEIN, RISC - Johannes Kepler University

Algorithms beyond the holonomic universe

A function is called holonomic or D -finite, if it satisfies a linear differential equation with polynomial coefficients. It is well known that D -finite functions are closed under certain operations that can be implemented and used to automatically prove identities on D -finite functions. We introduced the notion of DD -finite functions that satisfy linear differential equations with D -finite function coefficients and showed that also this class satisfies several closure properties. This construction can be iterated, yielding D^n -finite functions. In this talk, we report on what is currently known about these extensions. This is joint work with Antonio Jiménez Pastor.

MICHAEL WALLNER, TU Wien

Compacted binary trees and minimal automata admit stretched exponentials

A compacted binary tree is a directed acyclic graph encoding a binary tree in which common subtrees are factored and represented only once. We show that the number of such trees of size n is equal to

$$\Theta(n! 4^n e^{3a_1 n^{1/3}} n^{3/4}),$$

where $a_1 \approx -2.338$ is the largest root of the Airy function. Our approach involves bijections to enriched Dyck paths, two-parameter recurrences, induction, asymptotically tight bounds, and adapted Newton polygons. This method also allows to enumerate minimal DFAs recognizing a finite binary language. This is joint work with Andrew Elvey Price and Wenjie Fang.