Cycles in Planar Graphs

(Org: **Abhinav Shantanam** (Simon Fraser University) and/et **Carol T. Zamfirescu** (Ghent University))

XIAONAN LIU, Georgia Institute of Technology

Number of Hamiltonian cycles in planar triangulations

Hakimi, Schmeichel, and Thomassen conjectured in 1979 that if G is a 4-connected planar triangulation with n vertices then G contains at least $2(n-2)(n-4)$ Hamiltonian cycles, with equality if and only if G is a double wheel. Alahmadi, Aldred, and Thomassen recently proved that there are exponentially many Hamiltonian cycles in 5-connected planar triangulations. We consider 4-connected planar n-vertex triangulations G that do not have too many separating 4-cycles or have minimum degree 5. We show that if G has $O(n/\log_2 n)$ separating 4-cycles then G has $\Omega(n^2)$ Hamiltonian cycles, and if $\delta(G) \geq 5$ then G has $2^{\Omega(n^{1/4})}$ Hamiltonian cycles.

ON-HEI SOLOMON LO, University of Science and Technology of China

Gaps in the cycle spectrum of polyhedral graphs

It was recently initiated by Merker to study whether every polyhedral graph must have a cycle length in some certain integer interval. For any positive integer k, define $f(k)$ (respectively, $f_3(k)$) to be the minimum integer $\geq k$ such that every 3-connected planar graph (respectively, 3-connected cubic planar graph) of circumference $\geq k$ has a cycle whose length is in the interval $[k, f(k)]$ (respectively, $[k, f_3(k)]$). We will describe how the values of $f(k)$ and $f_3(k)$ can be determined. This is a joint work with Qing Cui.

EMILY A. MARSHALL, Arcadia University

Hamiltonicity of planar graphs with a forbidden minor

Tutte proved that all 4-connected planar graphs are Hamiltonian, but it is well-known that 3-connected planar graphs are not necessarily Hamiltonian. In this talk, we discuss the Hamiltonicity of certain 3-connected planar graphs with forbidden minors.

JENS M. SCHMIDT, Hamburg University of Technology

The Isolation Lemma

A cycle C of a graph G is *isolating* if every component of $G - V(C)$ consists of a single vertex. We show that isolating cycles in polyhedral graphs can be extended to larger ones: every isolating cycle C of length $6 \leq |E(C)| < \left\lfloor \frac{2}{3}(|V(G)| + 4) \right\rfloor$ implies an isolating cycle C' of larger length that contains $V(C)$. By "hopping" iteratively to such larger cycles, we obtain a powerful and very general inductive motor for proving long cycles and computing them (in quadratic runtime).

This is joint work with Jan Kessler.

ABHINAV SHANTANAM, Simon Fraser University

Pancyclicity in 4-connected planar graphs

A graph on n vertices is said to be pancyclic if, for each $k \in \{3, ..., n\}$, it contains a cycle of length k. Following Bondy's meta-conjecture that almost any nontrivial condition on a graph which implies Hamiltonicity also implies pancyclicity, Malkevitch conjectured that a 4-connected planar graph is pancyclic if it contains a cycle of length 4. We show that, for any edge e in a 4-connected planar graph G, there exist at least $\lambda(n-2)$ cycles of pairwise distinct lengths containing e, where $\lambda = 5/12$. We also show that λ can be $2/3$ at best. Joint work with Bojan Mohar.