Contributed Talks I

MACKENZIE CARR, Simon Fraser University

Digital Convexity in Cycles and Cartesian Products

Given a finite set V, a *convexity*, $\mathcal{C}$, is a collection of subsets of V that contains both the empty set and the set V and is closed under intersections. The elements of $\mathcal{C}$ are called *convex sets*. The *digital convexity* on the vertex set of a graph, originally introduced as a tool for processing digital images, is defined as follows: a subset $S \subseteq V(G)$ is digitally convex if, for every $v \in V(G)$, we have $N[v] \subseteq N[S]$ implies $v \in S$. Or, equivalently, S contains every vertex for which it is a local dominating set. In this talk, we discuss the use of cyclic binary strings and certain types of $n \times m$ binary arrays to enumerate the digitally convex sets of the k^{th} power of a cycle and of the Cartesian product of paths, $P_n \Box P_m$.

ALEXANDER KOLPAKOV, Université de Neuchâtel

Space vectors forming rational angles

We classify all sets of nonzero vectors in $\mathbb{R}^3$ such that the angle formed by each pair is a rational multiple of π. The special case of four-element subsets lets us classify all tetrahedra whose dihedral angles are multiples of π, solving a 1976 problem of Conway and Jones: there are 2 one-parameter families and 59 sporadic tetrahedra, all but three of which are related to either the icosidodecahedron or the B_3 root lattice. The proof requires the solution in roots of unity of a $W(D_6)$-symmetric polynomial equation with 105 monomials (the previous record was only 12 monomials). This is a joint work with Kiran Kedlaya, Bjorn Poonen, and Michael Rubinstein.

MATTHEW SULLIVAN, University of Waterloo

Simple Drawings of K_n from Rotation Systems

A complete rotation system on n vertices is a collection consisting of the cyclic permutations of the elements $[n]\backslash\{i\}$. If D is a drawing of a labelled graph, then a rotation at vertex v is the cyclic ordering of the edges at v. In particular, the collection of all vertex rotations of a simple drawing of K_n is a rotation system. Conversely, can we characterize when a complete rotation system can be represented as a simple drawing of K_n (a.k.a. realizable)? In 2011, Jan Kynčl published a proof using homotopy implying that if all 6 vertex rotation systems of an n vertex rotation system R_n are realizable, then R_n is realizable. In this talk, we will briefly review the full characterization of realizable rotation systems and see a combinatorial proof that if rotation systems of size 10 are realizable, then the associated n vertex rotation system is realizable.

JAMES TUITE, Open University, UK

The degree/geodecity problem for mixed graphs

The degree/girth problem asks for the smallest possible order of an undirected graph with given girth and minimum degree. In this talk we explore a new analogue of this problem for mixed graphs, i.e. graphs that contain both undirected edges and directed arcs. A mixed graph G is k-geodetic for some $k \geq 2$ if for any pair of vertices u, v of G there is at most one non-backtracking mixed path from u to v with length not exceeding k; we ask for the order of the smallest k-geodetic mixed graph with given undirected and directed degrees. An extremal mixed graph for this problem is called a geodetic cage. We present new lower bounds on the order of k-geodetic mixed graphs, results on their regularity and constructions of geodetic cages.

AARON WILLIAMS, Williams College

Constructing Universal Cycles for Fixed-Content Strings

Consider the circular string 123313321323 of length twelve. Its substrings of length three — $123, 233, 331, ..., 231, 312$ — encode the twelve permutations of the multiset $\{1, 2, 3, 3\}$ with the redundant final symbol omitted. We provide the first explicit construction of these fixed-content universal cycles, along with efficient algorithms that generate each successive symbol in amortized O(1)-time, regardless of the specific multiset of symbols.

When universal cycles of this type are decoded, the resulting order of strings (e.g. $1233, 2331, 3312, ..., 2313, 3123$) have a nice property: Successive strings differ by a prefix rotation of length n or $n-1$. We illustrate how this property can be used to speed-up exhaustive computations for the stacker crane problem, and other combinatorial problems whose candidate solutions can be represented by fixed-content strings.

Joint work with Joe Sawada (University of Guelph).