Signed Alternating-runs Enumeration in Classical Weyl Groups

The alternating-runs polynomial enumerates alternating runs in the symmetric group $S_n$. Three formulae are known for $R_{n,k}$, the number of permutations in $S_n$ with $k$ alternating runs, but all of them are complicated. We show that when alternating runs are enumerated with sign taken into account, one gets a neat formula. This has several consequences: we firstly get a near refinement of a result of Wilf on the exponent of $(1+t)$ that divides the alternating-runs polynomial in $A_n$, the alternating group. Other applications include a moment-type identity, and enumeration of alternating permutations in $A_n$. Similar results are obtained for the type B and type D Coxeter groups. This is a joint work with Krishnan Sivasubramanian.