The mean subtree order of graphs under edge addition

A subtree of a graph $G$ is a subgraph of $G$ that is a tree. The mean subtree order of $G$ is the average order of the subtrees of $G$. We conjecture that every non-complete graph $G$ contains a pair of nonadjacent vertices $u$ and $v$ such that adding the edge between $u$ and $v$ increases the mean subtree order, and we prove this conjecture in the case that $G$ is a tree. We discuss several related open problems and conjectures. This is joint work with Ben Cameron (University of Guelph).