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## The Metric Dimension of a Graph and its Variants - Part I

(Org: Shonda Dueck (University of Winnipeg))

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**FLORENT FOUCAUD**, University Clermont Auvergne, France

*Bounds on the order of a graph of given metric dimension and diameter: studies for standard graph classes*

It is easily seen that any graph of metric dimension  $k$  and diameter  $D$  has at most  $D^k + k$  vertices. This is almost never reached for general  $k$  and  $D$ ; a tight bound (still exponential) was derived by Hernando, Mora, Pelayo, Seara and Wood in 2011. However, for many graph classes, a polynomial bound holds. We discuss such bounds for trees, interval graphs, permutation graphs, planar graphs, etc. One of the tools that is helpful here is the notion of distance- $V$ - $C$  dimension. Joint work with Laurent Beaudou, Peter Dankelmann, Michael Henning, Arnaud Mary, George Mertzios, Reza Naserasr, Aline Parreau, Petru Valicov.

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**DOROTA KUZIAK**, Universidad de Cadiz, Spain

*The strong metric dimension of a graph*

A vertex  $w$  *strongly resolves* a pair  $u, v$  of vertices of a connected graph  $G$  if there exists some shortest  $w - u$  path containing  $v$  or some shortest  $w - v$  path containing  $u$ . A set  $S$  of vertices is a *strong metric generator* for  $G$  if every pair of vertices of  $G$  is strongly resolved by some vertex of  $S$ . The smallest cardinality of a strong metric generator for  $G$  is the *strong metric dimension* of  $G$ . We shall present exact values or sharp bounds for the strong metric dimension of cactus graphs and some product graphs.

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**TERO LAIHONEN**, Turku University, Finland

*On Vertices Belonging to Every Metric Basis*

A set  $S \subseteq V(G)$  is a resolving set in a graph  $G$  if for any pair  $u, v \in V(G)$  there exists  $s \in S$  such that  $d(u, s) \neq d(v, s)$ . A metric basis is a resolving set of the smallest possible cardinality. It is known that there are graphs where some vertices must belong to every metric basis. We call these vertices *basis forced vertices*. In this talk, we give, for example, bounds on the size of a graph with  $k$  basis forced vertices.

This is a joint work with Anni Hakanen, Ville Junnila and Ismael Yero.

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**ELIZABETH MARITZ**, University of the Free State, South Africa

*On the partition dimension of circulant graphs*

Let  $\Pi = \{S_1, S_2, \dots, S_k\}$  be an ordered partition of the vertex set  $V(G)$  of a graph  $G$ . The *partition representation* of a vertex  $v \in V(G)$  with respect to  $\Pi$  is the  $k$ -tuple  $r(v|\Pi) = (d(v, S_1), d(v, S_2), \dots, d(v, S_k))$ . If every pair of distinct vertices have distinct partition representations with respect to  $\Pi$ , then  $\Pi$  is a *resolving partition* for  $G$ . The cardinality of a smallest resolving partition of  $G$  is called the *partition dimension* of  $G$ . We present exact values and bounds on the partition dimension for undirected and directed circulant graphs.

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**ISMAEL GONZALEZ YERO**, Universidad de Cadiz, Spain

*Comparing the metric and edge metric dimensions of graphs*

Given a connected graph  $G$ , the cardinality of a smallest set of vertices that uniquely identifies all the (vertices or edges, resp.) of  $G$ , through a vector of distances to such set of vertices, is the (metric or edge metric, resp.) dimension of  $G$ . We shall present in this talk some comparisons between metric and edge metric dimension of graphs. Specifically, that for every  $r, t \geq 2$ , with  $r \neq t$ , there is  $n_0$ , such that for every  $n \geq n_0$  there exists a graph  $G$  of order  $n$  with metric dimension  $r$  and edge metric dimension  $t$ .