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Uniqueness of the Gibbs measure for the 4-state anti-ferromagnetic Potts model on the regular tree

We show that the 4-state anti-ferromagnetic Potts model with interaction parameter $w \in (0, 1)$ on the infinite $(d + 1)$ -regular tree has a unique Gibbs measure for all $w \geq 1 - \frac{4}{d+1}$ and $d \geq 4$. This is tight since it is known that there are multiple Gibbs measures when $0 \leq w < 1 - \frac{4}{d+1}$ and $d \geq 4$. The transition from having a unique to several Gibbs measures is closely connected to phase transitions in statistical physics.

Our method also gives a new proof of the uniqueness of the Gibbs measure for the 3-state Potts model on the $(d + 1)$ -regular tree for $w \geq 1 - \frac{3}{d+1}$ when $d \geq 3$ and for $w \in (0, 1)$ when $d = 2$.