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Counting lattice walks confined to cones

The study of lattice walks confined to cones is a lively topic in enumerative combinatorics, and has witnessed rich developments in the past 20 years. Typically, one is given a finite set of steps S in Z^d, and a cone C in R^d. Exactly $|S|^n$ walks of length n start from the origin and take their steps in S. But how many remain in the cone C?

One of the motivations for studying such questions is that such walks encode many objects in discrete mathematics, statistical physics, probability theory, among other fields.

In the past 20 years, several approaches have been combined to understand how the choice of the steps and of the cone influence the nature of the counting sequence $a(n)$, or of the the associated series $A(t) = \sum a(n)t^n$. Is $A(t)$ rational, algebraic, or solution of a differential equation? This is now completely understood when C is the first quadrant of the plane and S only consists of "small" steps. This "simple" case involves tools coming from an attractive variety of fields: algebra on formal power series, complex analysis, computer algebra, differential Galois theory. Much remains to be done, for other cones and sets of steps.