What is the least surface area of a body $B$ whose $\mathbb{Z}^n$ translations tile $\mathbb{R}^n$? The isoperimetric inequality gives the bound $\Omega(\sqrt{n})$, and remarkably Kindler et al. showed that this is achievable.

In this work, we consider permutation-symmetric tilings. We show that in this case the answer is $\Theta(n/\sqrt{\log n})$.

Our work is motivated by the study of strong versions of the parallel repetition theorem, which if true would have significant applications. Unfortunately, strong parallel repetition fails in general [Raz]. Our result suggests there may be important special cases where it still applies.

Joint work with Mark Braverman.