Arithmetic Combinatorics - Part II

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GABRIEL CONANT, University of Cambridge

Quantitative stable arithmetic regularity in arbitrary finite groups

In 2018, Terry and Wolf showed that any stable subset of a finite abelian group can be efficiently approximated by cosets of a subgroup whose index is bounded exponentially in the approximation and stability constants. At the same time, in joint work with Pillay and Terry, we proved a version of this for arbitrary finite groups using model theoretic techniques, with stronger qualitative features, but no quantitative bounds. Here I will discuss a new effective proof of our result, which yields quantitative bounds for arbitrary finite groups, and improves the bound in Terry & Wolf's result from exponential to polynomial.

ARTURO RODRIGUEZ FANLO, University of Oxford

On metric approximate subgroups

In 2011, using model theory, Hrushovski found a connexion between finite approximate subgroups and Lie groups. This result, known as the Lie model Theorem, was the starting point used to finally give a complete classification of finite approximate subgroups by Breuillard, Green and Tao. In this talk we will see a generalization of the Lie model Theorem to metric approximate subgroups, where small thickenings are also allowed.

WEIKUN HE, Korea Institute for Advanced Study

Sum-product estimates in semisimple algebras and random walks on the torus

I will present some results in the spirit of Bourgain's discretized sum-product theorem, but for general semisimple algebras over the real numbers. Then I will highlight an application to ergodic theory. More precisely, I will explain how these sum-product estimates are used in proving new results on the equidistribution of linear random walks on the torus. This talk is based on joint works with Nicolas de Saxcé.

SIMON MACHADO, University of Cambridge

Approximate Subgroups, Meyer Sets and Arithmeticity

Approximate lattices are discrete subsets of locally compact groups that are aperiodic but nonetheless exhibit long range order. In abelian groups, these subsets correspond to the so-called quasi-crystals and were given a precise structure theory by Meyer: he showed that they are projections of arithmetic subsets in higher dimension. We will discuss how tools originating from ergodic theory, aperiodic order and the structure of finite approximate subgroups enable us to generalise at once Meyer's theorem and Margulis' arithmeticity. In particular we show that approximate lattices in $SL_n(K),\ n \geq 3$ consist of matrices with coefficients in the set of Pisot numbers.

CHIEU-MINH TRAN, University of Notre Dame

A nonabelian Brunn-Minkowski inequality

Henstock and Macbeath asked in 1953 whether the Brunn-Minkowski inequality can be generalized to nonabelian locally compact groups; questions in the same line were also asked by Hrushovski, McCrudden, and Tao. We obtain here such an inequality and prove that it is sharp for helix-free locally compact groups, which includes real linear algebraic groups, Nash groups, semisimple Lie groups with finite center, solvable Lie groups, etc. As an application, we obtain a characterization of sets with nearly minimal measure expansion in noncompact locally compact groups, answering another question by Tao.