
Algebraic combinatorics I
(Chair/Président: **Alexander Garver** (UQAM))

LOGAN CREW, University of Pennsylvania

A Deletion-Contraction Relation for the Chromatic Symmetric Function

Richard Stanley introduced the *chromatic symmetric function* of a graph $G = (V, E)$ as a generalization of the chromatic polynomial $\chi_G(x)$, defined by

$$X_G(x_1, x_2, \dots) = \sum_{\substack{\kappa: V(G) \rightarrow \mathbb{Z}^+ \\ uv \in E(G) \implies \kappa(u) \neq \kappa(v)}} \prod_{v \in V(G)} x_{\kappa(v)}$$

In this talk, we extend the definition of X_G to include graphs (G, w) with a vertex-weight function $w : V(G) \rightarrow \mathbb{Z}^+$. This allows us to generalize the deletion-contraction relation of the chromatic polynomial. I will show how we can use this relation to derive alternate proofs of classical properties of X_G by proving them for the class of all vertex-weighted graphs. We also mention similarities with other functions on vertex-weighted graphs, including the W -polynomial of Noble and Welsh.

This is joint work with Sophie Spirkl.

HIRANYA KISHORE DEY, IIT Bombay, India

Gamma positivity of the Excedance based Eulerian polynomial in positive elements of Classical Weyl Groups

The classical Eulerian polynomials $A_n(t)$ are known to be gamma positive. Define the positive Eulerian polynomial $AExc_n^+(t)$ as the polynomial obtained when we sum excedances over the alternating group. We show that $AExc_n^+(t)$ is gamma positive iff $n \geq 5$ and $n \equiv 1 \pmod{2}$. When $n \geq 4$, and $n \equiv 0 \pmod{2}$ we show that $AExc_n^+(t)$ can be written as a sum of two gamma positive polynomials. Similar results are shown when we consider the positive type-B and type-D Eulerian polynomials. Finally, we show gamma positivity results when we sum excedances over derangements with positive and negative sign. Our main result is that the polynomial obtained by summing excedance over a conjugacy class indexed by λ is gamma positive.

This talk is based on joint work with Krishnan Sivasubramanian.

ORLI HERSCOVICI, Technion - Israel Institute of Technology

p, q -deformed Touchard polynomials and statistics on set partitions

A two-parameter deformation of the Touchard polynomials, based on the NEXT q -exponential function of Tsallis, defines two statistics on set partitions. The generating function of classical Touchard polynomials is a composition of two exponential functions. By applying analysis of a combinatorial structure of the deformed exponential function, we establish explicit formulae for both statistics. Moreover, those statistics let us formulate a new combinatorial proof of some known combinatorial identities.

YOTSANAN MEEMARK, Chulalongkorn University

Zero Divisor Graphs of Finite Chain Rings

A finite chain ring is a finite commutative ring such that for each ideals I and J , we have $I \subseteq J$ or $J \subseteq I$. In this talk, we first discuss zero divisor graphs over finite chain rings. We determine their rank, determinant and eigenvalues by using reduction graphs. Moreover, we extend the work to zero divisor graphs over finite quotient rings of unique factorization domains by using a combinatorial method and we find upper and lower bounds for the largest eigenvalue.

AMARPREET RATTAN, Simon Fraser University

Factorizations of canonical full cycle, k -parking functions and cacti.

It is a well-known result of Kreweras that the inversion enumerator for trees is equal to the area enumerator for parking functions. On the other hand, Biane and Stanley found that, in the symmetric group, minimal length factorizations of the canonical full cycle into transpositions are connected to parking functions via a remarkably simple bijection. We recently found that these well-known relationships between factorizations, parking functions and trees could be generalized and significantly refined. Here we present a new approach to this refinement in which the major index on trees plays a central and natural role. We find new connections between minimal length factorizations of the canonical full cycle into k -cycles and inversions and non-inversions in cacti. These results generalize earlier work of Shin, as well as work of Yan connecting the area enumerator of k -parking functions to k -inversions on forests. This is joint work with J. Irving.