
LOGAN CREW, University of Pennsylvania

A Deletion-Contraction Relation for the Chromatic Symmetric Function

Richard Stanley introduced the *chromatic symmetric function* of a graph $G = (V, E)$ as a generalization of the chromatic polynomial $\chi_G(x)$, defined by

$$X_G(x_1, x_2, \dots) = \sum_{\substack{\kappa: V(G) \rightarrow \mathbb{Z}^+ \\ uv \in E(G) \implies \kappa(u) \neq \kappa(v)}} \prod_{v \in V(G)} x_{\kappa(v)}$$

In this talk, we extend the definition of X_G to include graphs (G, w) with a vertex-weight function $w : V(G) \rightarrow \mathbb{Z}^+$. This allows us to generalize the deletion-contraction relation of the chromatic polynomial. I will show how we can use this relation to derive alternate proofs of classical properties of X_G by proving them for the class of all vertex-weighted graphs. We also mention similarities with other functions on vertex-weighted graphs, including the W -polynomial of Noble and Welsh.

This is joint work with Sophie Spirkl.