On approximation of $H$-coloring

The minimum cost homomorphism problem (MinHOM) is a natural optimization problem for homomorphisms to a fixed (di)graph $H$ (a.k.a $H$-coloring). Given an input (di)graph $G$, with a cost associated with mapping any vertex of $G$ to any vertex of $H$, one seeks to minimize the sum of costs of the assignments over all homomorphisms of $G$ to $H$.

We are interested in the approximation of MinHOM within a constant factor. We give a dichotomy classification for approximating the MinHOM($H$) when $H$ is a graph. For digraphs, we provide constant factor approximation algorithms for two important classes of digraphs, namely bi-arc digraphs and k-arc digraphs. Specifically, we have:

1-Dichotomy for Graphs: MinHOM($H$) has a $2 |V(H)|$-approximation algorithm if graph $H$ is a bi-arc graph, otherwise, it is inapproximable;

2- MinHOM($H$) has a $|V(H)|^2$-approximation algorithm if $H$ is a bi-arc digraph;

3- MinHOM($H$) has a $|V(H)|^2$-approximation algorithm if $H$ is a k-arc digraph.