A lower bound on the size of linear sets on a projective line of finite order

A set $S$ of points of the finite projective space $\text{PG}(\mathbb{F}_{q}^{n+1})$ is a linear set of rank $t$ if $S = \{ \langle u \rangle_{\mathbb{F}} \mid u \in U \}$ for some set $U \subset \mathbb{F}_{q}^{n+1}$ that is a $t$-dimensional vector space over a subfield of $\mathbb{F}_{q}$.

The following result was obtained jointly with Geertrui Van de Voorde: An $\mathbb{F}_{q}$-linear set of rank $k \leq n$ in $\text{PG}(1, q^{n})$ which contains at least one point of weight one, contains at least $q^{k-1} + 1$ points.

This result, its connection with direction problems in affine spaces, and some applications will be discussed.