The positivity of trees

The chromatic polynomial was generalized to the chromatic symmetric function by Stanley in 1995, and started two research directions. Firstly, does the chromatic symmetric function distinguish trees, and secondly when can the chromatic symmetric function be written as a positive linear combination of elementary symmetric functions.

In this talk we meld these two directions and discover when the chromatic symmetric function of trees is not a positive linear combination of elementary symmetric functions, using a beautiful but overlooked result of Wolfgang III from 1997.

This is joint work with Samantha Dahlberg and Adrian She.