Online Bipartite Matching with Amortized $O(\log^2 n)$ Replacements

In Online Bipartite Matching with Replacements, all the vertices on one side of the bipartition are given, and the up to $n$ vertices on the other side arrive one-by-one with all their incident edges. The goal is to maintain a maximum matching while minimizing the number of changes (replacements) to the matching. We show that the greedy algorithm that always takes the shortest augmenting path (SAP) from the newly inserted vertex uses at most amortized $O(\log^2 n)$ replacements per insertion, approaching the $\Omega(\log n)$ lower bound. The previous best strategy [Bosek, Leniowski, Sankowski, Zych, FOCS 2014] achieved amortized $O(\sqrt{n})$ replacements.