Colourings and homomorphisms

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DEBRA BOUTIN, Hamilton College

Geometric Homomorphisms and the Geochromatic Number

A geometric graph $\overline{G}$ is a simple graph G together with a fixed straightline drawing in the plane. A geometric homomorphism is a homomorphism of the underlying abstract graphs that also preserves edge crossings. Like the chromatic number of an abstract graph, the geochromatic number is defined using homomorphisms: the geochromatic number of a geometric graph $\overline{G}$ is the smallest integer n so that there is a geometric homomorphism from $\overline{G}$ to some geometric realization of K_n. This talk will consider conditions under which we can find or bound the geometric chromatic number of a geometric graph.

RICHARD BREWSTER, Thompson Rivers University

The complexity of signed graph homomorhpisms

A *signed graph* (G, Σ) is a graph G where each edge is given a sign, positive or negative; $\Sigma \subseteq E(G)$ denotes the set of negative edges. Central to signed graphs is the operation of *resigning* at a vertex.

An *s-homomorphism* from (G, Σ) to (H, Π) is a vertex map that preserves edges and their signs after possibly resigning at some vertices of G. We give a simple classification of the $\mathbf{P}/\mathbf{NP}$-complete dichotomy for the (H, Π)-colouring problem and a direct proof of the result.

This is joint work with Foucaud, Hell, Naserasr, and Siggers.

CHRISTOPHER DUFFY, University of Saskatchewan

Colourings, Simple Colourings, and a Connection to Bootstrap Percolation

Define a k-colouring of a mixed graph to be a homomorphism to a k-vertex (complete) mixed graph. A simple colouring of a mixed graph is a homomorphism to a reflexive mixed graph. Intuitively one would expect the simple chromatic number of a family of mixed graphs to be less than the chromatic number – surely allowing adjacent vertices to have the same colour will allow us to use fewer colours. In this talk we explore cases where our intuition fails us using via a connection between simple colouring and a directed bootstrap percolation process.

JOHN GIMBEL, University of Alaska

Bounds on the fractional chromatic number of a graph.

Given a graph G and I, the family of independent sets in G, a function f:$I \rightarrow$ [0, ∞) is a fractional coloring if for each vertex v, the inequality $1 \leq \sum_{\text{v} \in I}$ f(I) holds. Further, the weight of f is w(f) $= \sum_{\text{I} \in I}$ f(I). The fractional chromatic number, $\chi_f(G)$, is the minimum weight of all fractional colorings. We develop methods for producing upper bounds on χ_f. This leads to simple proofs for theorems similar to Grotzch's 3-colorability theorem and the five color theorem for planar graphs.

JAE-BAEK LEE, Kyungpook National University

Reconfiguring Reflexive Digraphs

For a fixed graph H, the recolouring problem for H-colouring(i.e. homomorphisms to H) ask: given an irreflexive graph G and two H-colourings ϕ and ψ of G, is it possible to transform ϕ into ψ by changing the colour of one vertex at a time such that all intermediate mappings are H-colourings? For a reflexive graph G, one requires the extra condition that the colour must be changed to a neighbour of its current colour.