Bootstrap Percolation

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Polluted Bootstrap Percolation

On the $d-$dimensional integer lattice, make every site independently occupied with probability p, closed with probability q, and open otherwise. Then iteratively occupy all open sites with at least threshold θ occupied neighbors. Closed and occupied sites never change. When both p and q are small, what relation between them guarantees that most sites eventually become occupied? The talk will describe recent progress on this problem, which is joint work with A. Holroyd and D. Sivakoff.

LIANNA HAMBARDZUMYAN, McGill University

Polynomial method and graph bootstrap percolation

We introduce a simple method for proving lower bounds for the size of the smallest percolating set in a certain graph bootstrap process. We apply this method to determine the sizes of the smallest percolating sets in multidimensional tori and multidimensional grids (in particular hypercubes). The former answers a question of Morrison and Noel [MN], and the latter provides an alternative and simpler proof for one of their main results.

IVAILO HARTARSKY, École normale supérieure de Lyon

The second term for two-neighbour bootstrap percolation in two dimensions

In two-neighbour bootstrap percolation on $[n]^2$ vertices with at least two infected neighbours are iteratively infected. Initial infections are binomial with parameter p. Motivation comes from connections to the Ising and Fredrickson-Andersen models.

We seek the location $p_c(n)$ of the transition of complete infection. Successively better bounds were given in a founding work of Aizenman and Lebowitz, a breakthrough of Holroyd and, a decade ago, by Gravner and Holroyd and Gravner, Holroyd and Morris. In this work we improve on GHM to match the bound of GH, thus proving

$$p_c(n) = \frac{\pi^2}{18 \log n} - \frac{\Theta(1)}{(\log n)^{3/2}}.$$

Joint with Robert Morris.

DAVID SIVAKOFF, The Ohio State University

Bootstrap percolation on Cartesian products of lattices with Hamming graphs

Spread of signals on graphs with community structure has attracted interest in the mathematical literature recently. We model the spread of signals using the bootstrap percolation dynamics with integer threshold $r > 1$, and take our graph to be the Cartesian product of d_1 integer lattices (or cycles) with d_2 (finite) complete graphs. Thus, each "community" consists of individuals determined by d_2 characteristics, and two individuals within a community only communicate if they have all but one of the characteristics in common. Between communities, communication is between like individuals that are also neighbors in the lattice $\mathbb{Z}^{d_1}$. In collaboration with Janko Gravner.