Design Theory

IREN DARIJANI, Memorial University of Newfoundland

The chromatic index of block intersection graphs of Steiner triple systems

A Steiner triple system of order v is a pair $(V, \mathcal{B})$, where V is a set of v points and $\mathcal{B}$ is a collection of 3-subsets of points, called blocks, such that every 2-subset of points occurs in exactly one block. The block intersection graph of a Steiner triple system with block set $\mathcal{B}$ is the graph with $\mathcal{B}$ as its vertex set such that two vertices are adjacent if and only if their associated blocks are not disjoint. The chromatic index of a graph G is the least number of colours that enable each edge of G to be assigned a single colour such that adjacent edges never have the same colour. In this talk, we will discuss the chromatic index of block intersection graphs of Steiner triple systems, with particular emphasis on cyclic Steiner triple systems. Additional new results might also be presented.

ARAS ERZURUMLUOGLU, University of Ottawa

Hamiltonian Properties of 2-Block-Intersection Graphs of Twofold Triple Systems

(joint work with David Pike)

A *balanced incomplete block design* (BIBD(v, k, λ)) $(V, \mathcal{B})$ is a combinatorial design in which (i) $|V| = v$, (ii) for each block $B \in \mathcal{B}$, $|B| = k$, and (iii) each 2-subset of V occurs in precisely λ blocks of $\mathcal{B}$. A BIBD$(v, 3, 2)$ is a *twofold triple system* (TTS(v)).

Given a combinatorial design $\mathcal{D}$ with block set $\mathcal{B}$, the *i-block-intersection graph* (i-BIG) of $\mathcal{D}$ is the graph having $\mathcal{B}$ as its vertex set, where two vertices $B_1 \in \mathcal{B}$ and $B_2 \in \mathcal{B}$ are adjacent if and only if $|B_1 \cap B_2| = i$.

Recently we have settled the spectrum of TTSs with connected non-Hamiltonian 2-BIGs, as well as the spectrum of TTSs with Hamiltonian 2-BIGs. In this talk I will present some of the techniques that were used to obtain these results.

KEVIN HALASZ, Simon Fraser University

Coloring Cayley tables

The chromatic number of a Latin square is defined as the minimum number of partial transversals needed to cover all of its cells. It has been conjectured that every Latin square L of order n satisfies $\chi(L) \leq n + 2$. If true, this would resolve a longstanding conjecture of Brualdi and Stein that every Latin square has a near transversal. Restricting our attention to Cayley tables of finite groups, we prove two partial results. First, we construct $(n + 2)$-colorings for all Abelian groups, showing that the conjecture holds in this special case. Second, we give an upper bound for $\chi(L)$ that depends only on the order of the underlying group. This improves the best-known general upper bound from $2n$ to $\frac{3}{2}n$, while yielding an even stronger result in infinitely many cases.

SARA HERKE, The University of Queensland

Parity of MOLS

The useful notion of parity of permutations can be extended to Latin squares. Each Latin square has a row, column and symbol parity, but any two of these determines the third. We consider a direct generalization of the parity of a Latin square to the parity of a set of mutually orthogonal Latin squares (MOLS) and present some constraints on parity which are strictest in the case of MOLS corresponding to projective planes. Projective planes of order $n \equiv 2 \mod 4$, $n > 2$, are widely believed not to exist; our results give some insight as to why it is harder to build projective planes in this case.

SAMUEL SIMON, Simon Fraser University

Nonexistence Results for Systems of Linked Designs

A system of linked $(v,k,\lambda;n)$ designs satisfy $D_1D_2 = \nu D_3 + \mu(G - D_3)$ for parameters ν, μ in terms of v, k, λ. I will show that there are no such systems in a family of design parameters. Namely there are not three $(q^{d+1}(s+1), q^d s, q^d s - q^{2d}; q^{2d})$-designs via the McFarland construction satisfying the above linking property. No prior knowledge of design theory will be assumed, and the proof relies only on projections and some elementary number theory.