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Comparable pairs in families of sets

We consider a problem posed by Erdős and Daykin and Frankl in the early '80s. They asked for the maximum number of comparable pairs ($A \subseteq B$ or $B \subseteq A$) that can appear in a family of m subsets of $[n]$, a quantity we denote by $c(n, m)$. We first resolve an old conjecture of Alon and Frankl, showing that $c(n, m) = o(m^2)$ when $m = n^{\omega(1)}2^{n/2}$. We also obtain more accurate bounds for $c(n, m)$ for sparse and dense families, characterize the extremal constructions for certain values of m, and sharpen some other known results.

Joint work with Noga Alon, Shagnik Das, and Benny Sudakov.