This is a problem which came up in a class, and I could not determine the answer. Given a coloring of a graph, is there a pair of colors $i, j$ such that we can recolor only vertices with these 2 colors, while using only 1 new color $k$.

Note that this seems very similar to list coloring of bipartite graphs, which is NP-complete; however, in this problem there is the extra condition that all vertices may be colored $k$. 