Open Problems
(Chair/Président: **Brett Stevens** (Carleton University)
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BRIAN ALSPACH, University of Newcastle
Orthogonalizeable Groups

At the first CANADAM conference in 2007 I mentioned the following problem: Given a circulant digraph with connection set $S = \{s_1, \ldots, s_t\}$, is there a directed path of length t with one arc of each length from S? We now extend extend the problem to all finite groups. A group G is $orthogonalizeable$ if every Cayley digraph on G admits either an orthogonal directed path or an orthogonal directed cycle. This relates to Gordon's definition of sequenceable groups and we introduce the notion of a $sequenceable$ poset for attacking the following general problem: Determine which groups are orthogonalizeable.

PETER DUKES, University of Victoria
Avoiding long bicoloured cycles

This is an old problem of Roland Häggkvist which I believe deserves some promotion.

Over all n-edge-colourings of $K_{n,n}$, determine the minimum longest two-coloured cycle. Let's call this $f(n)$. I know that $f(n) \leq 182$ for all n and I wonder whether, for n mildly large, it is the case that

$$f(n) = \begin{cases} 4 & \text{if } n \text{ is a power of 2;} \\ 6 & \text{otherwise.} \end{cases}$$

KEVIN HALASZ, Simon Fraser University
Olson's Conjecture

In 1961, Erdős, Ginzberg, and Ziv proved that, for every sequence $g_1, \ldots, g_{2n-1}$ of elements of a finite solvable group G of order n, there exist indices $i_1, \ldots, i_n$ such that (writing the operation additively) $g_{i_1} + \cdots + g_{i_n} = 0$. In 1976, Olson eliminated the solvability condition, thereby extending the result to *all* finite groups, before conjecturing:

For any sequence of $g_1, \ldots, g_{2n-1}$ of elements of a finite group G, then there exist indices $i_1 < \cdots < i_n$ such that $g_{i_1} + \cdots + g_{i_n} = 0$.

Save some special cases, this problem remains open.

CHUNHUI LAI, Minnan Normal University
On Hajós conjecture

Hajós conjectured that every simple even graph on n vertices can be decomposed into at most $n/2$ cycles (see L. Lovasz, On covering of graphs, in: P. Erdos, G.O.H. Katona (Eds.), Theory of Graphs, Academic Press, New York, 1968, pp. 231 - 236). The survey article on this problem can be found in Chunhui Lai, Mingjing Liu[Some open problems on cycles, Journal of Combinatorial Mathematics and Combinatorial Computing 91 (2014), 51-64.] We do not think Hajós conjecture is true.

STANISLAW RADZISZOWSKI AND XIAODONG XU, Rochester Institute of Technology, NY and Academy of Sciences, Nanning, China
A question about growth of Ramsey numbers $R(3, k)$

For a graph G, the Ramsey number $R(3,G)$ is the smallest positive integer n such that every triangle-free graph of order n contains G in its complement. Here we focus on G being K_k or $K_k - e$. The asymptotics of $R(3,k) = R(3,K_k)$ was extensively studied and now it is quite well understood. It is known that

$$(1/4 + o(1))k^2/\log k \le R(3,k) \le (1 + o(1))k^2/\log k.$$

However, the difference $R(3,G) - R(3,H)$ for concrete "consecutive" H and G is still very difficult to estimate, starting already with rather small cases. In general, for K_k and $K_k - e$, all we know is the following:

(1) $3 \le R(3,K_k) - R(3,K_{k-1}) \le k$, easy old bounds,

(2) $R(3,K_{k-1}) \le R(3,K_k - e) \le R(3,K_k)$, trivial bounds,

(3) $4 \le R(3,K_{k+1}) - R(3,K_k - e)$ (Zhu-Xu-R 2015).

Many people tried to improve on some part of (1) or (2), to no avail. A relatively simple recent construction proving inequality (3) is an interesting step towards better understanding of both (1) and (2).

Problem: Improve over any of the inequalities in (1), (2) or (3), or their combination as (3) combines parts of (1) and (2).

ROBERT ŠÁMAL, Charles University in Prague

p-free flows

Let G be a digraph, M an abelian group, $\varphi : E(G) \to M$ a flow. We say φ is p-free if $\varphi(e_1) + \cdots + \varphi(e_k)$ is never zero for $1 \le k \le p$ and $e_1, \ldots, e_k \in E(G)$.

For $p = 1$ this is the nowhere-zero flow.

For $p = 2$ this is called antisymmetric flow.

Conjecture (Nešetřil, Šámal) For every $p \ge 1$ exists a K_p such that any orientation of an $(p+1)$-edge-connected graph has a p-free flow $\varphi : E(G) \to M$ where M is a group of order $\le K_p$.

True for $p = 1, 2$. (Jaeger; DeVos, Johnson and Seymour)

True for any p and $(2p+1)$-edge-connected graphs.

Open for other cases.

JERRY SPINRAD, Vanderbilt University

Local Improvement for Coloring

This is a problem which came up in a class, and I could not determine the answer. Given a coloring of a graph, is there a pair of colors i, j such that we can recolor only vertices with these 2 colors, while using only 1 new color k.

Note that this seems very similar to list coloring of bipartite graphs, which is NP-complete; however, in this problem there is the extra condition that all vertices may be colored k.

XIA ZHANG, University of Victoria, Canada; Shandong Normal University, China

The correlation between f-chromatic class and gc-chromatic class of a simple graph

An f-$coloring$, a g_c-$coloring$ of graph G is an edge-coloring such that the edge-induced subgraph of each color is a $(0,f)$-factor, (g,d)-factor, respectively.

Theorem 1. (S.L. Hakimi and O. Kariv, 1986) A simple graph G has either $\chi'_f(G) = \Delta_f(G)$ (f-class 1), or $\chi'_f(G) = \Delta_f(G) + 1$ (f-class 2), where $\Delta_f(G) = \max_{v \in V(G)} \{\lceil \frac{d(v)}{f(v)} \rceil\}$.

Theorem 2. (H. Song and G. Liu, 2005) A simple graph G has either $\chi'_{gc}(G) = \delta_g(G)$ (g_c-class 1), or $\chi'_{gc}(G) = \delta_g(G) - 1$ (g_c-class 2), where $\delta_g(G) = \min_{v \in V(G)} \{\lfloor \frac{d(v)}{g(v)} \rfloor\}$.

Problem. (G. Liu and X. Zhang) What kinds of simple graphs G have coincident classification results between f-coloring and g_c-coloring when $\{v : d(v) = \Delta_f(G)f(v)\} = \{v : d(v) = \delta_g(G)g(v)\}$?