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A question about growth of Ramsey numbers $R(3,k)$

For a graph G, the Ramsey number $R(3,G)$ is the smallest positive integer n such that every triangle-free graph of order n contains G in its complement. Here we focus on G being K_k or $K_k - e$. The asymptotics of $R(3,k) = R(3,K_k)$ was extensively studied and now it is quite well understood. It is known that

$$(1/4 + o(1))k^2/\log k \leq R(3,k) \leq (1 + o(1))k^2/\log k.$$

However, the difference $R(3,G) - R(3,H)$ for concrete "consecutive" H and G is still very difficult to estimate, starting already with rather small cases. In general, for K_k and $K_k - e$, all we know is the following:

(1) $3 \leq R(3,K_k) - R(3,K_{k-1}) \leq k$, easy old bounds,

(2) $R(3,K_{k-1}) \leq R(3,K_k - e) \leq R(3,K_k)$, trivial bounds,

(3) $4 \leq R(3,K_{k+1}) - R(3,K_k - e)$ (Zhu-Xu-R 2015).

Many people tried to improve on some part of (1) or (2), to no avail. A relatively simple recent construction proving inequality (3) is an interesting step towards better understanding of both (1) and (2).

Problem: Improve over any of the inequalities in (1), (2) or (3), or their combination as (3) combines parts of (1) and (2).