Cop and Robber Game in a Polygon

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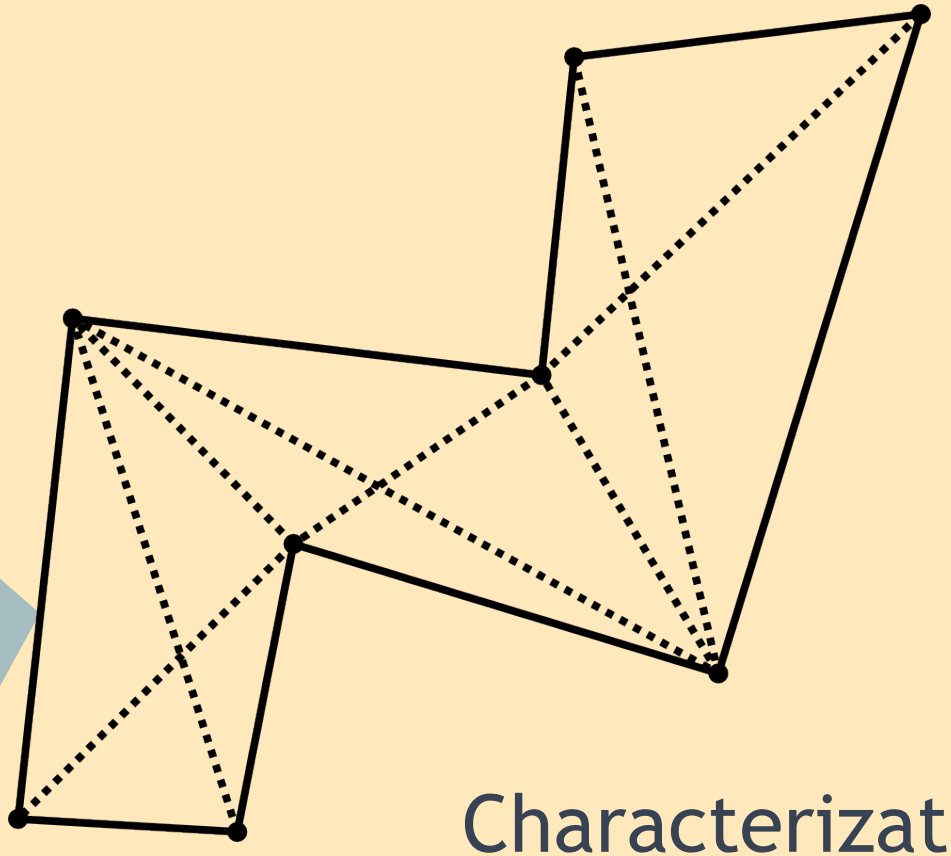
Cop and Robber Game in Polygons

Visibility Graphs

Dismantlable Graphs

Visibility Graph

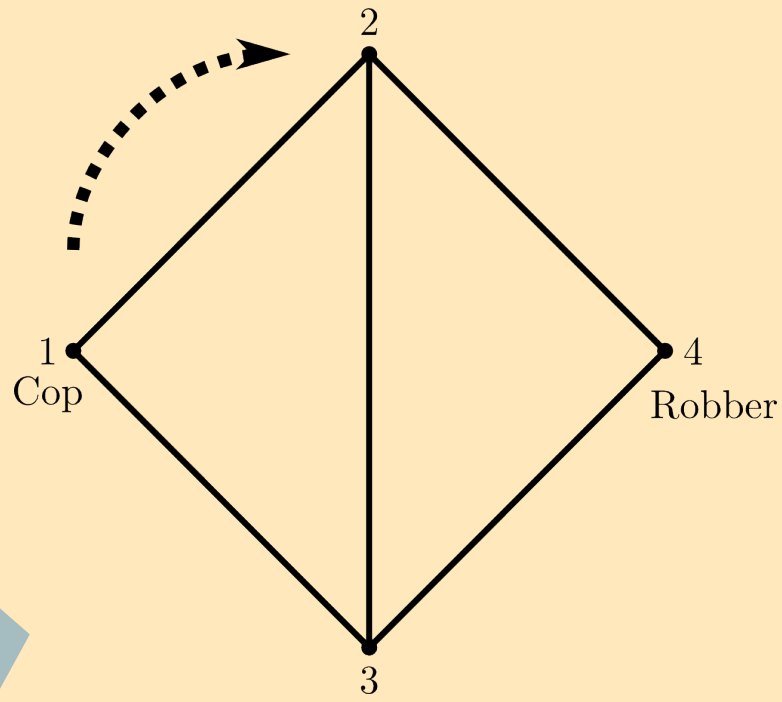


V = vertices

E = (u,v): u sees v

Characterization and recognition is still an open problem.

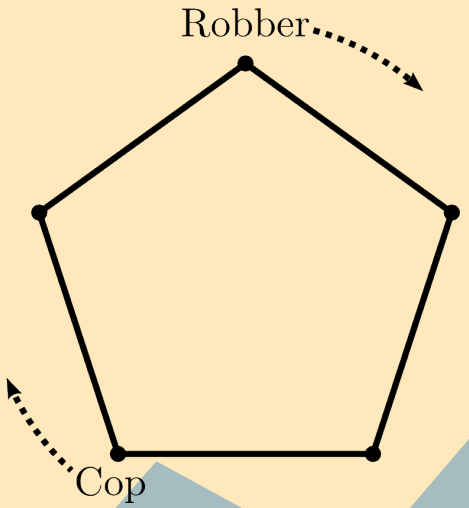
Cop and Robber Game



- cop & robber on vertices
- take turns, move on edges

Vertex 2 dominates vertex 4:

$N[4] \subseteq N[2]$

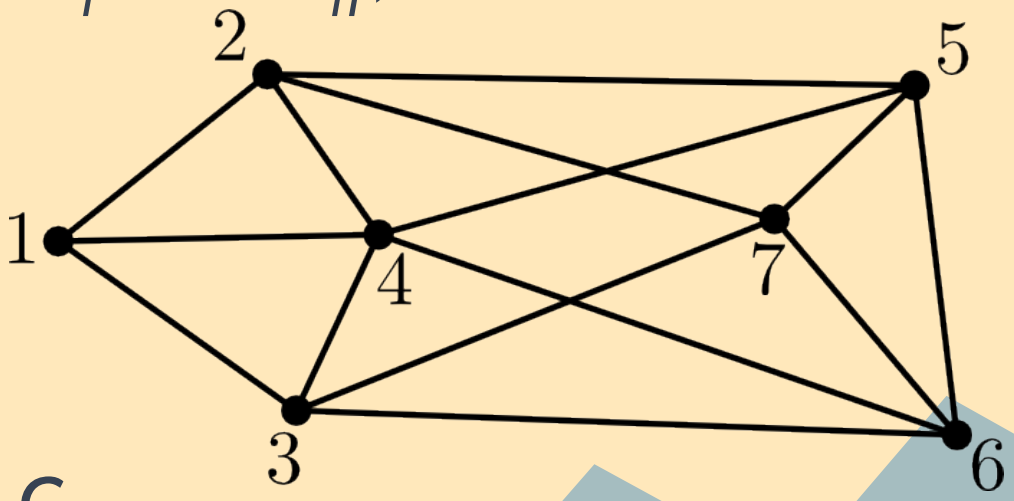


Dismantlable Ordering

Vertex ordering of $G=(V,E)$

$$v_1\ v_2 \ldots v_n$$

s.t. v_i dominated by some v_j , $j>i$ *in* G_i *(the subgraph of vertices* $v_i, \ldots, v_n$ *)*



1 dominated by 4

2 dominated by 5 in G_2

Dismantlability and Cop-win

Theorem by Nowakowski and Winkler 1982:

A graph $G=(V,E)$ is cop-win iff dismantlable.

- recognition in polynomial time
- cop optimal strategy in polynomial time
- number of turns bounded by n

2-Dismantlablity

The graph has at least 2 dominated vertices u_1 and u_2 and *G-u_1-u_2 is 2-dismantlable recursively.*

- number of turns bounded by $n/2$

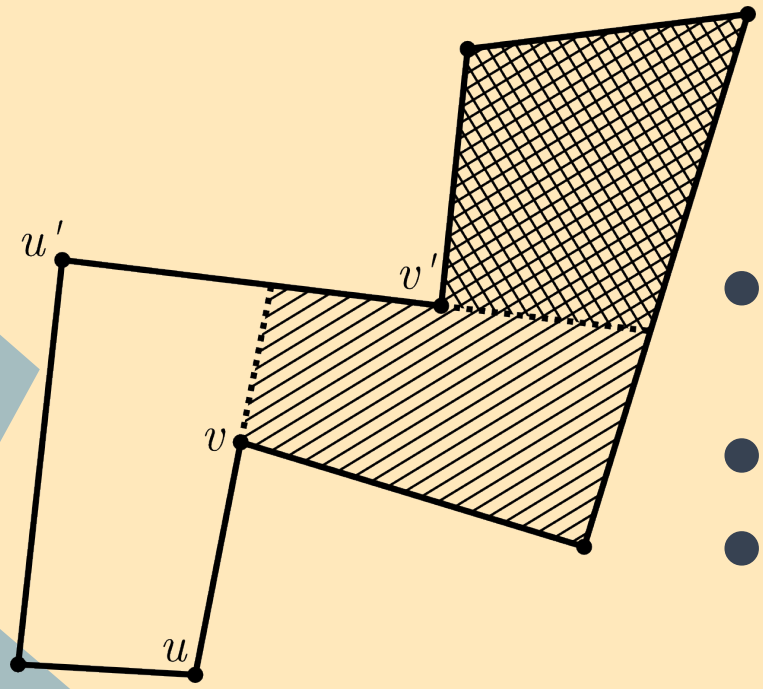
Visibility Graphs

Dismantlable Graphs

Visibility Graphs are Dismantlable

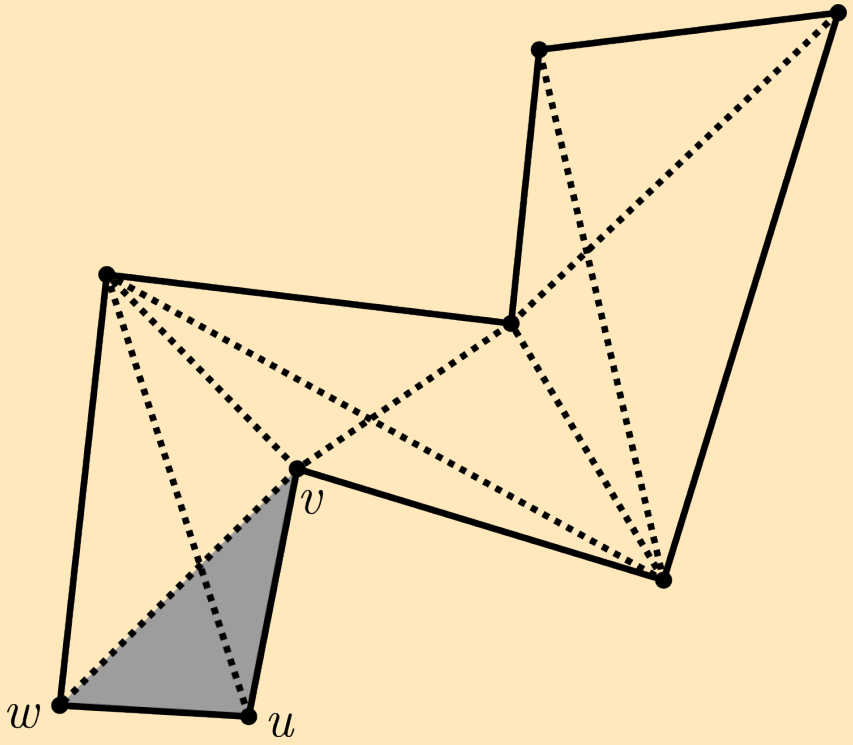
Proved by Aichholzer et al. 2011

Idea: A maximal pocket gives a dominated vertex



- pocket*(u,v)* maximal by containment.
- pocket*(u',v')* not maximal
- pocket*(u,v)* maximal $\Rightarrow$ *u* is dominated by *v*.

Visibility Graphs are Dismantlable

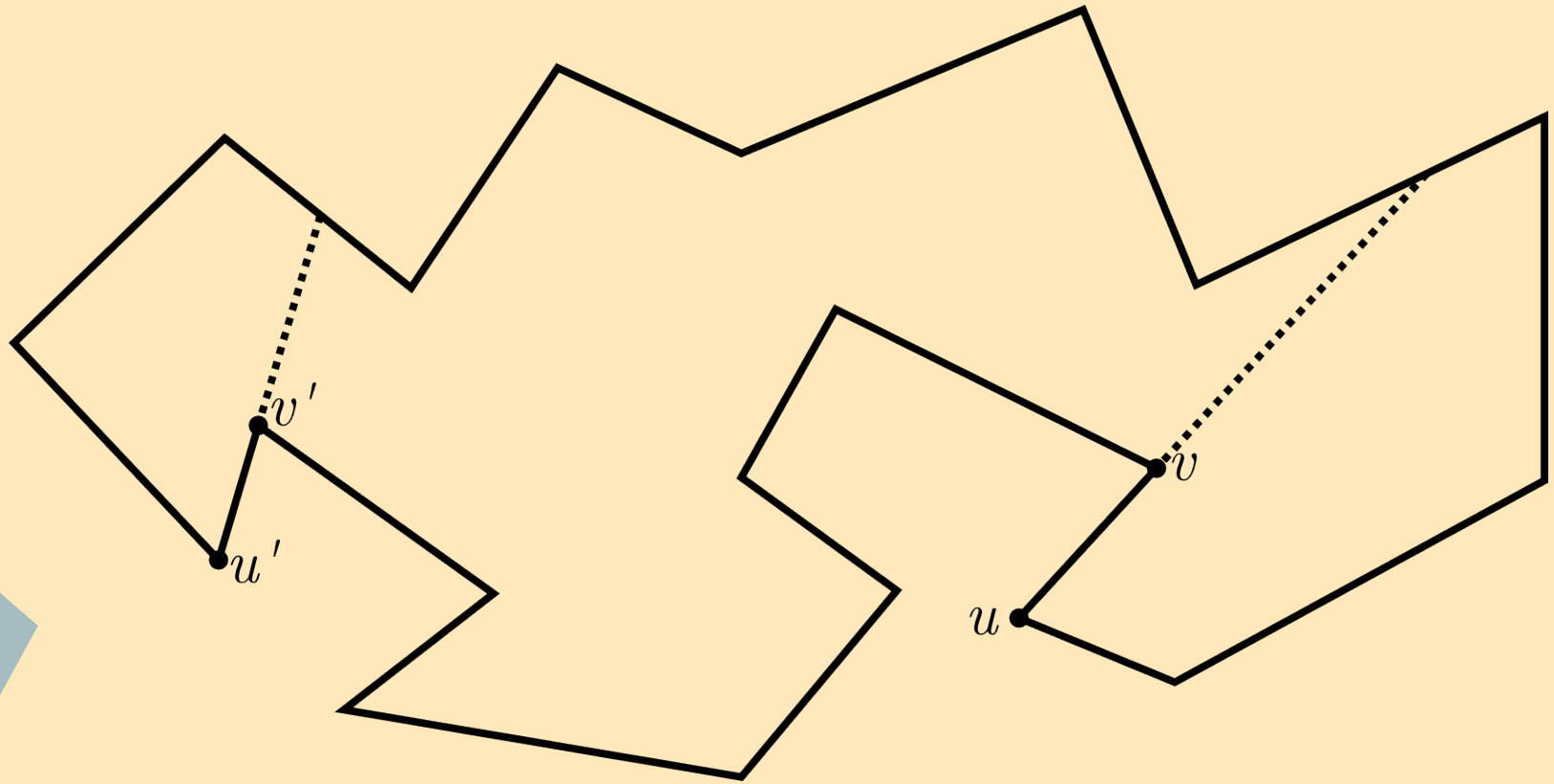


Remove *u* from visibility graph ~ Remove $\triangle uvw$ from polygon

Cops and Robbers on Visibility Graph

- always cop-win
- the cop may only use reflex vertices to win
- the game finishes after *r* turns (number of reflex vertices)

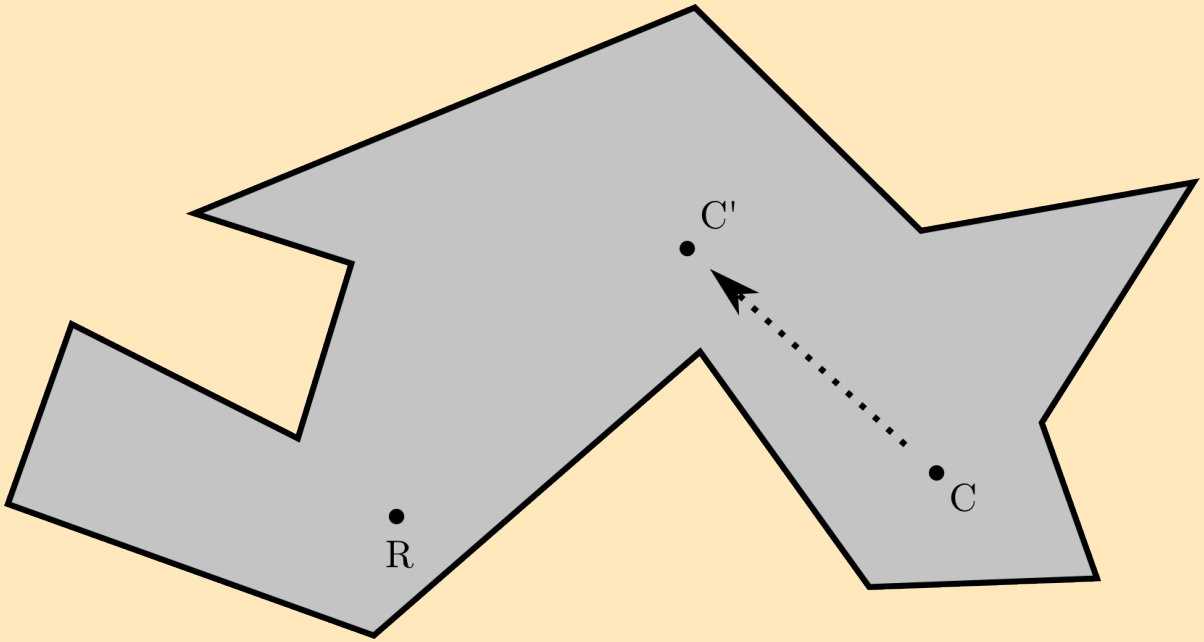
Visibility Graphs are 2-Dismantlable



u dominated by *v*

u' dominated by *v'*

Cops and Robbers in Polygons



- Take turns
- Move on straight line inside the polygon
- Have full information about each other's location
- Cop's goal is to capture the robber, i.e. move to robber's position.

Background on Pursuit Evasion

- Continuous vs discrete space (limited speed)
- Continuous vs discrete moves (man and lion problem)
- Capture vs see the evader.
- Group of pursuers vs a single pursuer.
- Full information vs partial/no information about the evader's position.

Our Result

Infinite Visibility Graph

- Vertex for every point inside polygon.
- Edge *(p,q)* if *p* sees *q*.

We Prove

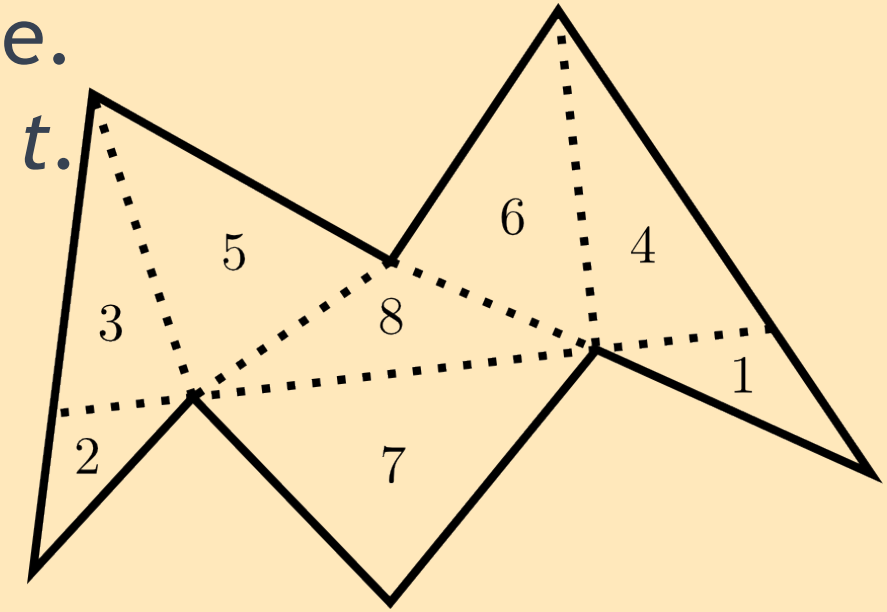
- The game is cop-win
- Using 2-dismantlable ordering of triangles in polygons.

Proof Idea

Successively remove dominated triangles.

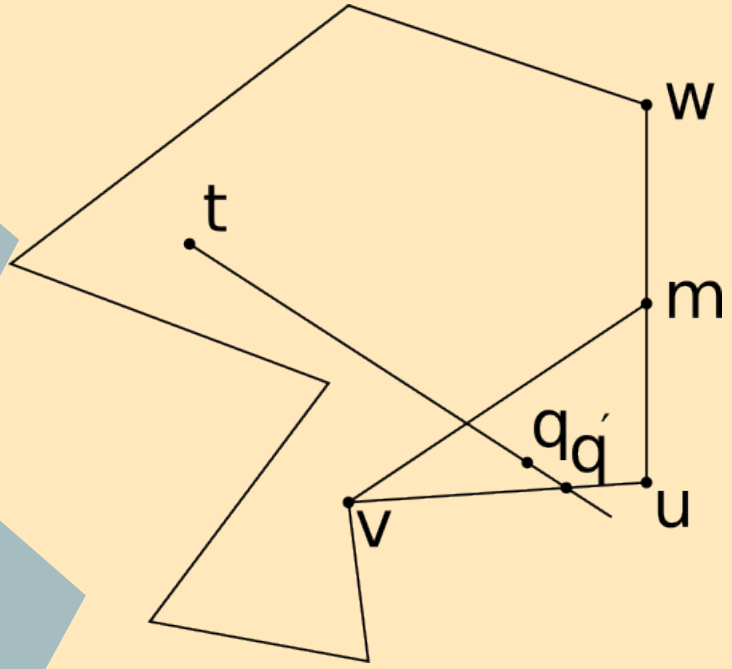
R dominated by *v* if all points $p \in R$ are dominated by *v* i.e.

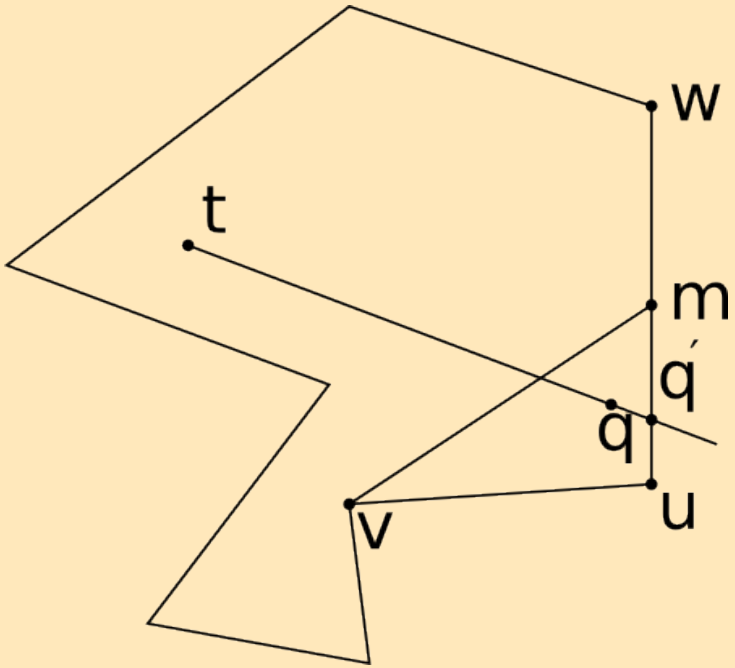
p sees *t* $\Rightarrow$ v sees *t*.



triangles' vertices are not necessarily polygon vertices.

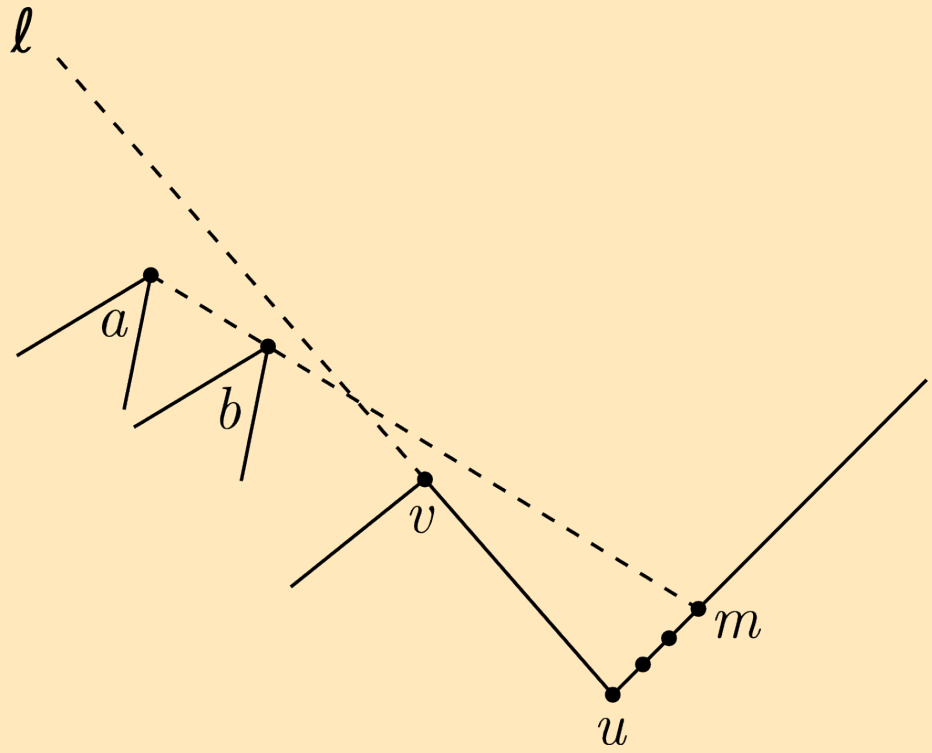
Dominated Triangle





If boundary of $\triangle vum$ is dominated by v, all points q inside the triangle are also dominated.

How to choose point *m*?

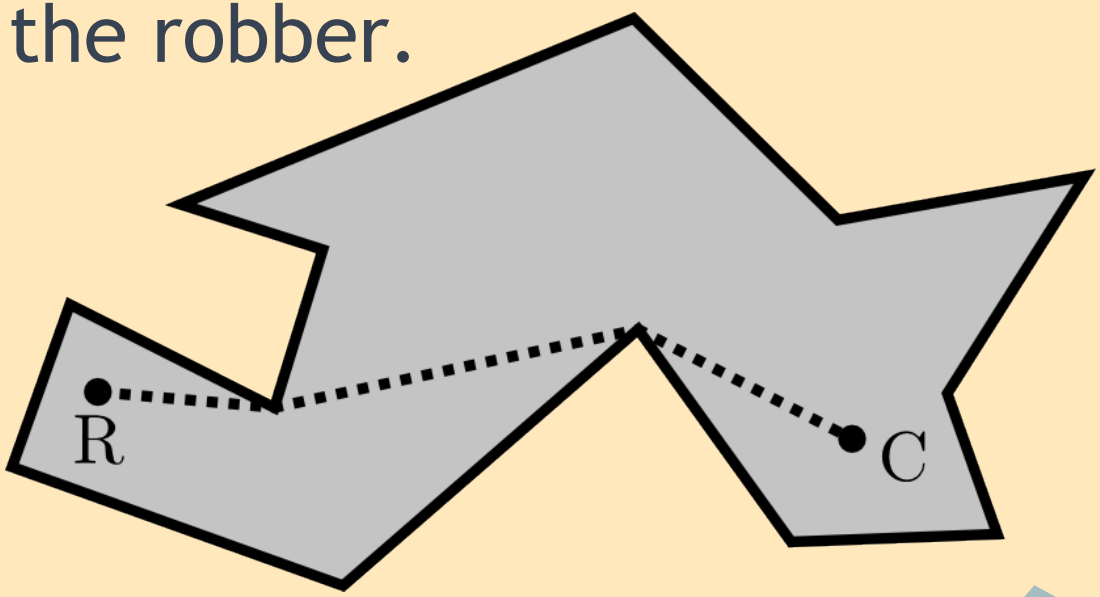


- Pocket*(u,v)* is maximal
- *m* is reflex collinear
- get $O(n^2)$ triangles.

Simple Strategy

We also showed a simple cop move rule to win:

The cop goes to the first step on the *shortest path* to the robber.

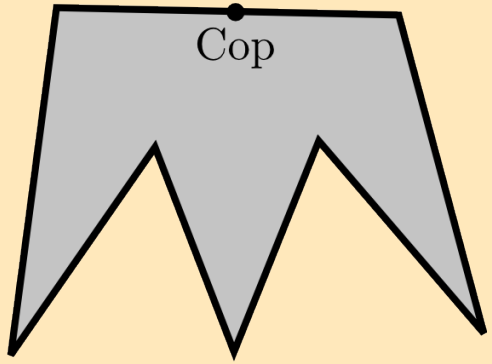


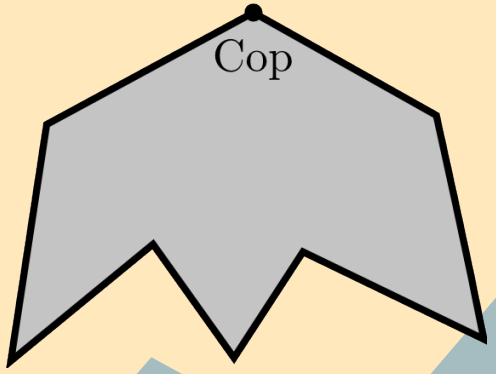
Conclusions

- Provide one answer to Hahn's question (2002) of finding non-trivial classes of infinite cop-win graphs
- One step into visibility graph characterization.
- The more natural way to model pursuit evasion problem than limited speed for real polygonal environments.

Open Questions

- How hard is to find the optimal cop strategy
  - polynomial when the cop is limited to vertices (maybe boundary)





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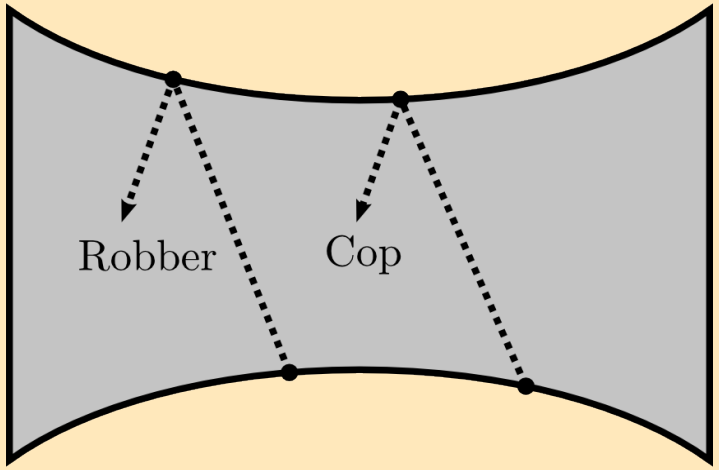
- Game in curvy environments
  - no winning strategy for cop moving on boundary



Open Questions

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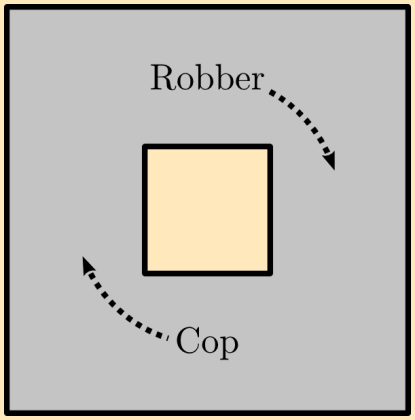
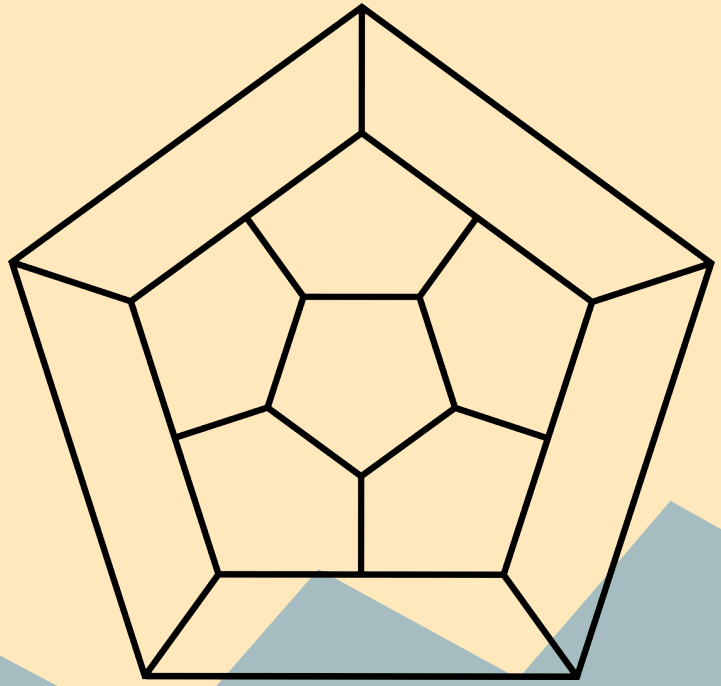
  - polynomial when the cop is limited to vertices (maybe boundary)
- Game in curvy environments
  - no winning strategy for cop moving on boundary
- Number of sufficient cops in polygons with holes
  - 3 cops are needed in some situations (based on the plannar graph by Aigner and Fromme (1984) that needs 3 cops)





THANK YOU