The Minimum Number of Distinct Eigenvalues of a Graph

Representing Saskatchewan: hard to spell, easy to draw

University of Regina's Discrete Math Group: Bahman Ahmadi, Fatemeh Alinaghipour, Robert Bailey, Michael Cavers, Shaun Fallat, Karen Meagher, Shahla Nasserasr

University of Regina, Saskatchewan

CanaDAM, June 2013

The Discrete Math Group

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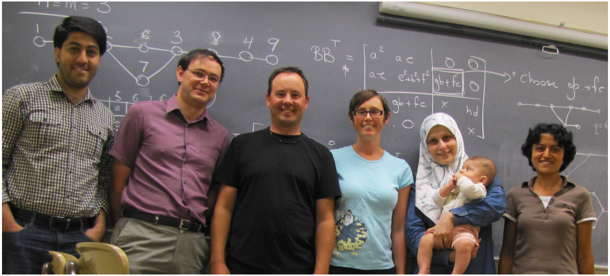
Our Website is:

`http://www.math.uregina.ca/~kmeagher/DMRG.html`



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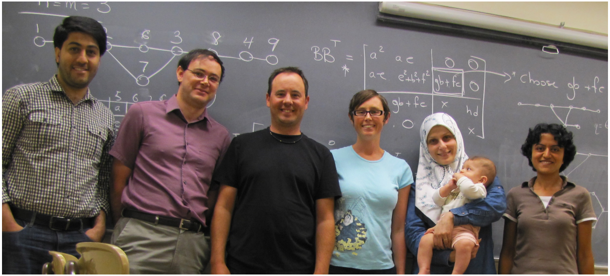

Publications:

1. Minimum number of distinct eigenvalues of graphs.
2. The minimum rank of universal adjacency matrices.
3. Generalized covering designs and clique coverings.

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3. How does changing G change $q(G)$?

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$$q(G) \leq mr(G) + 1$$

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5. $q(C_n) = \left\lceil \frac{n}{2} \right\rceil$.

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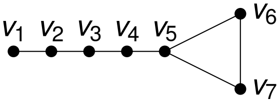

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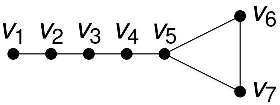

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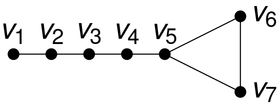

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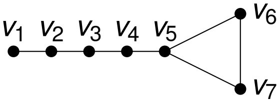

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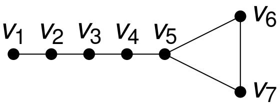

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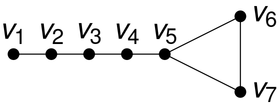

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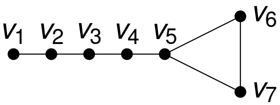

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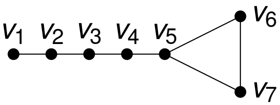

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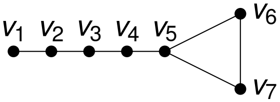

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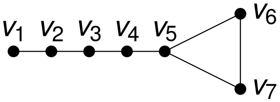

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6. Thus $q(A) \geq 6$.

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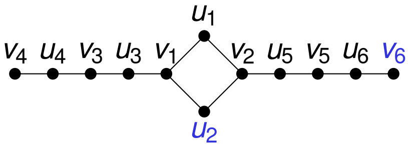
2. If T is a tree, then $q(T) \geq \operatorname{diam}(T) + 1$.

Another Example



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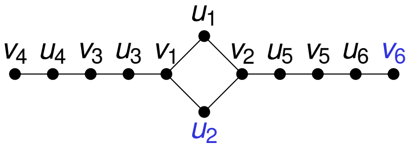

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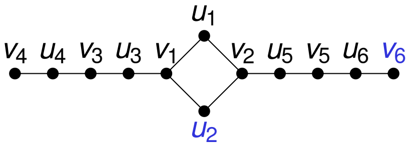

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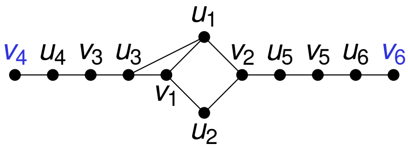

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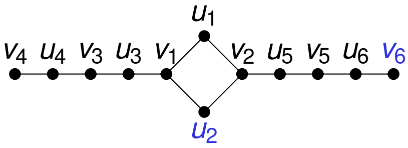

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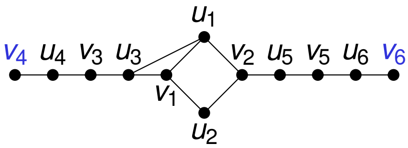

This graph has at least 10 distinct eigenvalues.

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- $q(K_n \backslash e) = 2$ where $n > 3$ and e is any edge.

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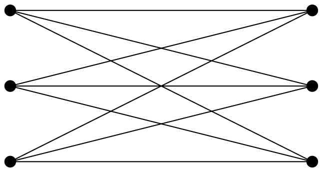

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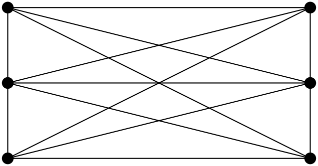

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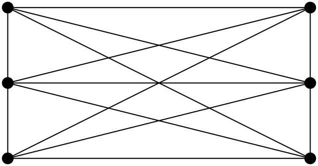

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Theorem

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- $Q^2 = I$, which implies that Q has eigenvalues 1 and -1.

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2. Set $R = I - P$ then R is an invertible "M-matrix", so it has a square root.

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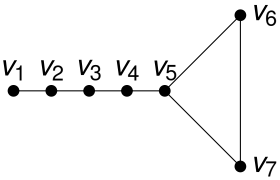
- they are only the “parallel paths”.

Two Graphs with a Large Number of Distinct Eigenvalues



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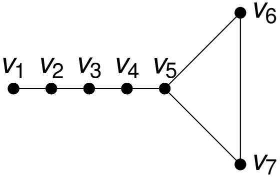

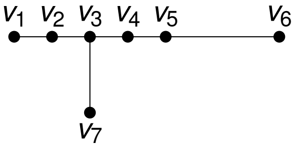

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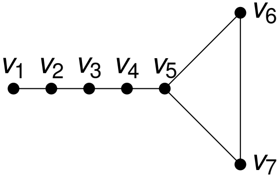

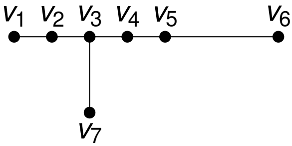

We suspect that these are all the graphs with
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