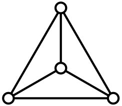
K_4-based and $\overline{C_6}$-based planar bricks

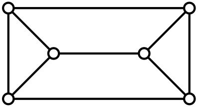
Nishad Kothari
nkothari@math.uwaterloo.ca

(joint work with U. S. R. Murty)

June 10, 2013 @ CanaDAM

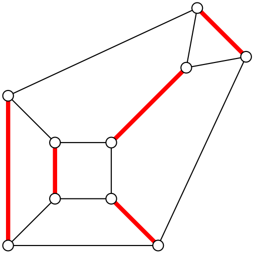


K_4

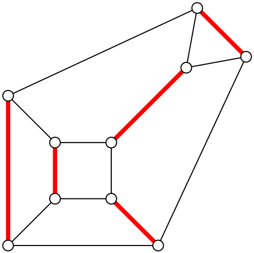


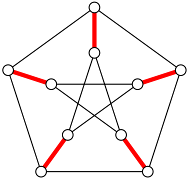
$\overline{C_6}$

Matching Covered Graphs



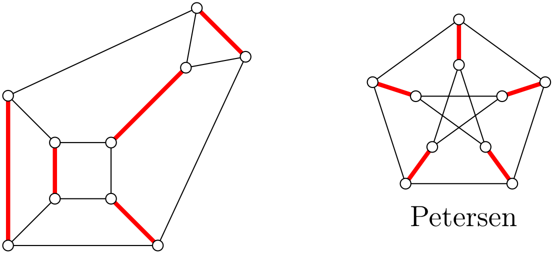
Matching Covered Graphs





Petersen

Matching Covered Graphs

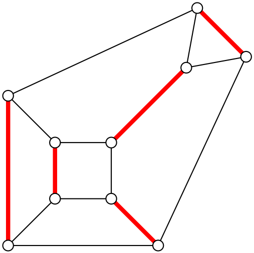


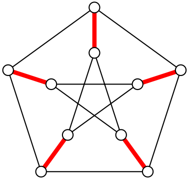
Petersen

Definition (Matching Covered Graph)

A connected graph with at least two vertices is *matching covered* if each of its edges lies in some perfect matching.

Matching Covered Graphs





Petersen

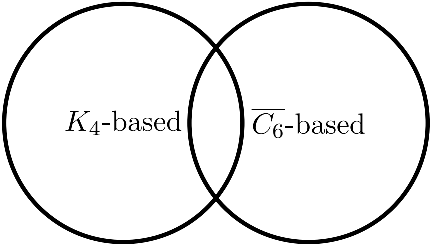
Definition (Matching Covered Graph)

A connected graph with at least two vertices is *matching covered* if each of its edges lies in some perfect matching.

For example, any 2-connected cubic graph is matching covered.

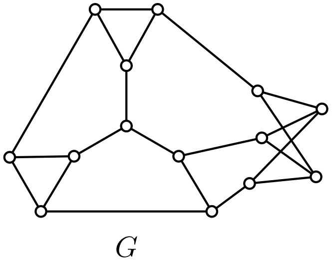
A classification of nonbipartite matching covered graphs

A classification of nonbipartite matching covered graphs

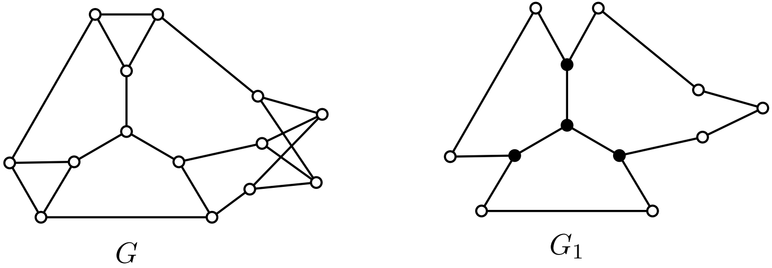


Building a matching covered graph

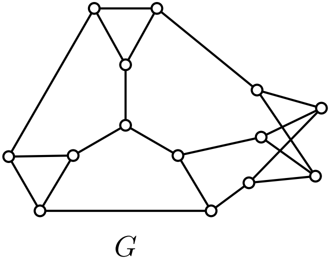
Building a matching covered graph

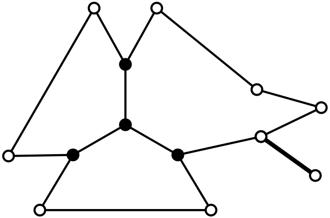


Building a matching covered graph

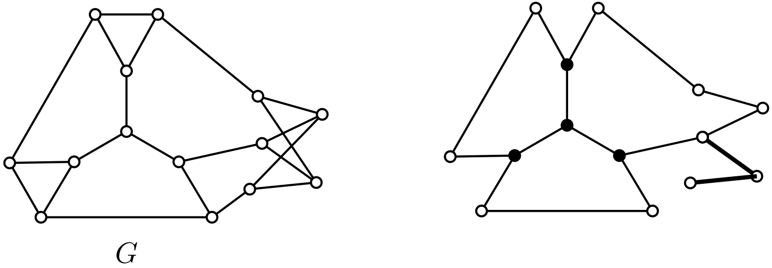


Building a matching covered graph

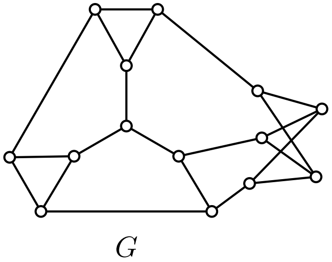


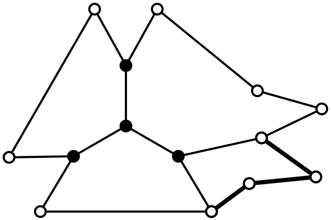


Building a matching covered graph

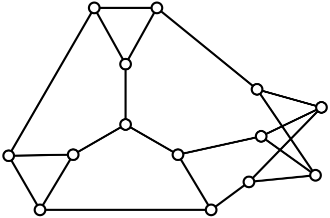


Building a matching covered graph

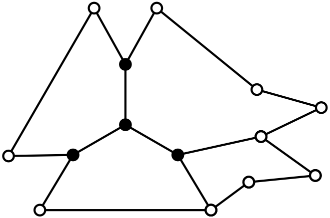




Building a matching covered graph

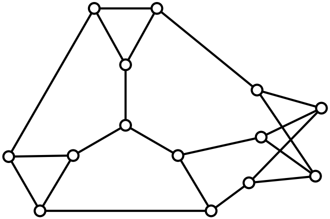


G

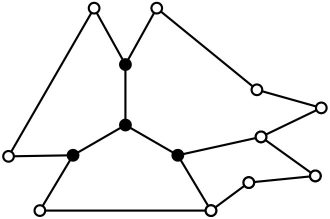


G_2

Building a matching covered graph



G



G_2

Ear decomposition

Definition (Single and double ears)

A *single ear* of a graph is a path of odd length whose each internal vertex (if any) has degree two in the graph.

Ear decomposition

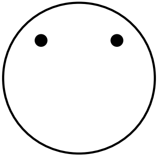
Definition (Single and double ears)

A *single ear* of a graph is a path of odd length whose each internal vertex (if any) has degree two in the graph. A pair of vertex-disjoint single ears is called a *double ear*.

Ear decomposition

Definition (Single and double ears)

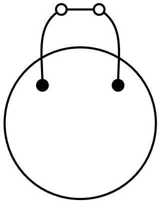
A *single ear* of a graph is a path of odd length whose each internal vertex (if any) has degree two in the graph. A pair of vertex-disjoint single ears is called a *double ear*.



Ear decomposition

Definition (Single and double ears)

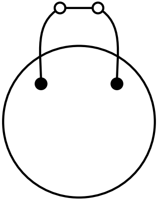
A *single ear* of a graph is a path of odd length whose each internal vertex (if any) has degree two in the graph. A pair of vertex-disjoint single ears is called a *double ear*.

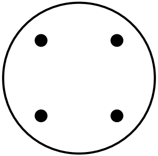


Ear decomposition

Definition (Single and double ears)

A *single ear* of a graph is a path of odd length whose each internal vertex (if any) has degree two in the graph. A pair of vertex-disjoint single ears is called a *double ear*.

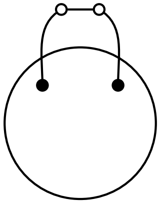


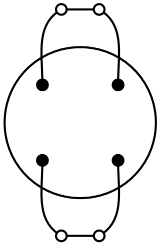


Ear decomposition

Definition (Single and double ears)

A *single ear* of a graph is a path of odd length whose each internal vertex (if any) has degree two in the graph. A pair of vertex-disjoint single ears is called a *double ear*.





Ear decomposition

Definition (Ear decomposition)

An *ear decomposition* of a matching covered graph G is a sequence $G_1 \subset G_2 \subset ... \subset G_r$ of matching covered subgraphs of G such that:

Ear decomposition

Definition (Ear decomposition)

An *ear decomposition* of a matching covered graph G is a sequence $G_1 \subset G_2 \subset ... \subset G_r$ of matching covered subgraphs of G such that:

1. for each i such that $1 \leq i \leq r-1$, G_{i+1} is the union of G_i and exactly one single ear or double ear of G_{i+1}, and

Ear decomposition

Definition (Ear decomposition)

An *ear decomposition* of a matching covered graph G is a sequence $G_1 \subset G_2 \subset \ldots \subset G_r$ of matching covered subgraphs of G such that:

1. for each i such that $1 \leq i \leq r-1$, G_{i+1} is the union of G_i and exactly one single ear or double ear of G_{i+1}, and
2. $G_r := G$.

Ear decomposition theorem

Theorem (Lovász)

Every nonbipartite matching covered graph G has an ear decomposition $G_1 \subset G_2 \subset ... \subset G_r$ such that G_1 is either:

Ear decomposition theorem

Theorem (Lovász)

Every nonbipartite matching covered graph G has an ear decomposition $G_1 \subset G_2 \subset \ldots \subset G_r$ such that G_1 is either:

1. *a bi-subdivision of K_4*

Ear decomposition theorem

Theorem (Lovász)

Every nonbipartite matching covered graph G has an ear decomposition $G_1 \subset G_2 \subset \ldots \subset G_r$ such that G_1 is either:

1. *a bi-subdivision of K_4 (we say G is K_4-based), or*

Ear decomposition theorem

Theorem (Lovász)

Every nonbipartite matching covered graph G has an ear decomposition $G_1 \subset G_2 \subset \ldots \subset G_r$ such that G_1 is either:

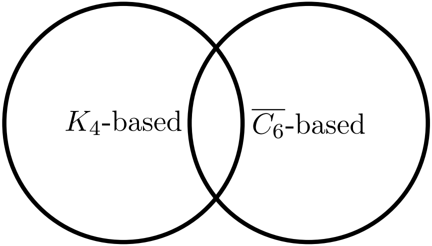
1. *a bi-subdivision of K_4 (we say G is K_4-based), or*
2. *a bi-subdivision of $\overline{C_6}$*

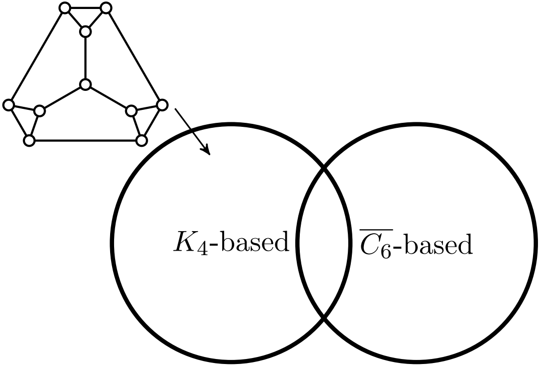
Ear decomposition theorem

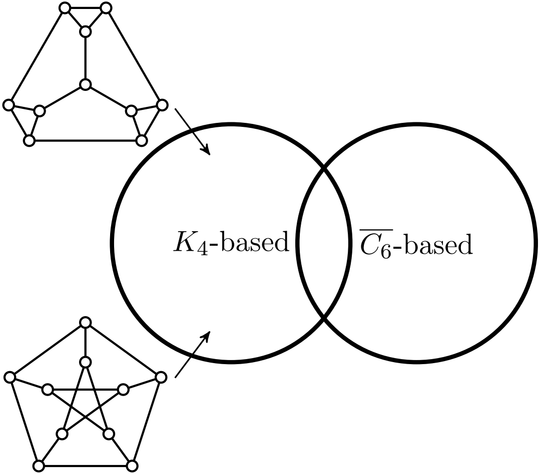
Theorem (Lovász)

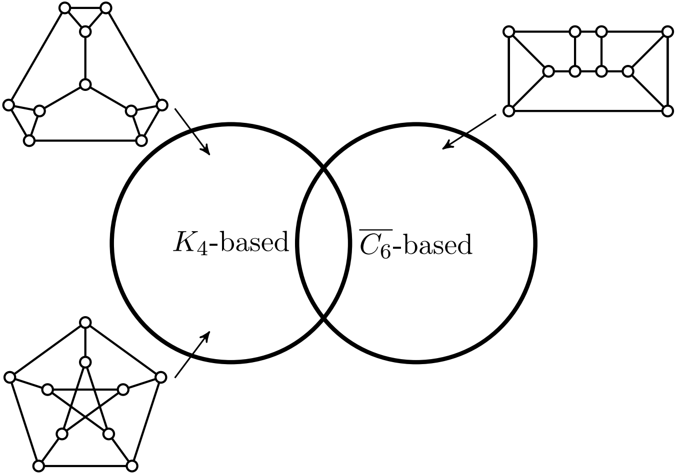
Every nonbipartite matching covered graph G has an ear decomposition $G_1 \subset G_2 \subset \ldots \subset G_r$ such that G_1 is either:

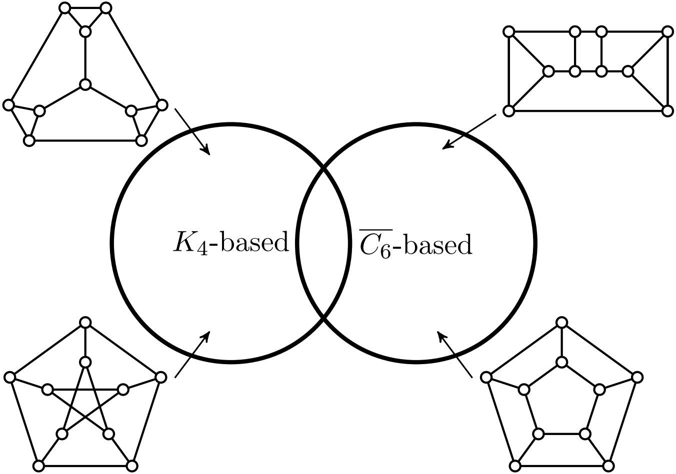
1. *a bi-subdivision of K_4 (we say G is K_4-based), or*
2. *a bi-subdivision of $\overline{C_6}$ (we say G is $\overline{C_6}$-based).*

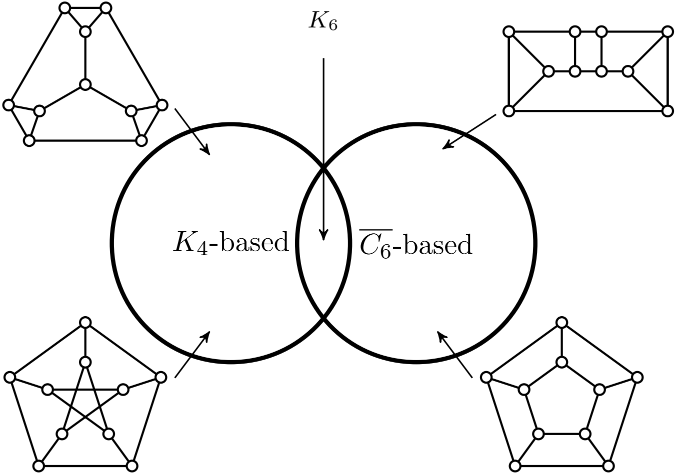

K_4-based
$\overline{C_6}$-based


K_4-based
$\overline{C_6}$-based


K_4-based
$\overline{C_6}$-based


K_4-based
$\overline{C_6}$-based


K_4-based
$\overline{C_6}$-based


K_6
K_4-based
$\overline{C_6}$-based

Bricks

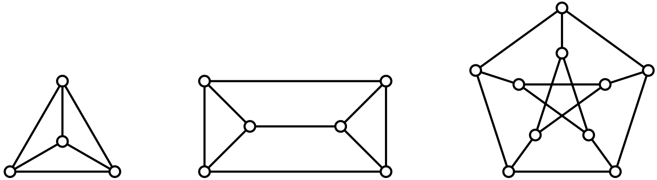
Definition (Brick)

A 3-connected bicritical graph is called a *brick*.
(These are special nonbipartite matching covered graphs.)

Bricks

Definition (Brick)

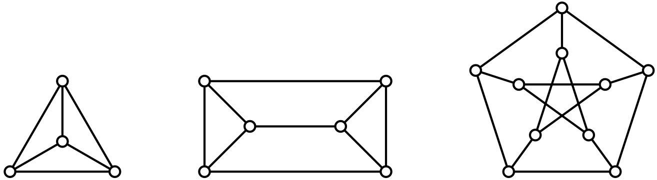
A 3-connected bicritical graph is called a *brick*.
(These are special nonbipartite matching covered graphs.)



Bricks

Definition (Brick)

A 3-connected bicritical graph is called a *brick*.
(These are special nonbipartite matching covered graphs.)



Corollary

Every brick is either K_4-based or $\overline{C_6}$-based (or possibly both).

A necessary condition for a planar brick to be K_4-based

Let G be a plane matching covered graph which is K_4-based.

A necessary condition for a planar brick to be K_4-based

Let G be a plane matching covered graph which is K_4-based.

Then G has an ear decomposition $G_1 \subset G_2 \subset \ldots \subset G_r$ such that G_1 is a bi-subdivision of K_4.

A necessary condition for a planar brick to be K_4-based

Let G be a plane matching covered graph which is K_4-based.

Then G has an ear decomposition $G_1 \subset G_2 \subset \ldots \subset G_r$ such that G_1 is a bi-subdivision of K_4.

Note that K_4 has exactly four odd faces, and so does G_1.

A necessary condition for a planar brick to be K_4-based

Let G be a plane matching covered graph which is K_4-based.

Then G has an ear decomposition $G_1 \subset G_2 \subset ... \subset G_r$ such that G_1 is a bi-subdivision of K_4.

Note that K_4 has exactly four odd faces, and so does G_1.

We conclude that G has at least four odd faces.

A necessary condition for a planar brick to be K_4-based

Let G be a plane matching covered graph which is K_4-based.

Then G has an ear decomposition $G_1 \subset G_2 \subset ... \subset G_r$ such that G_1 is a bi-subdivision of K_4.

Note that K_4 has exactly four odd faces, and so does G_1.

We conclude that G has at least four odd faces.

A necessary condition for a planar brick to be K_4-based

Let G be a plane matching covered graph which is K_4-based.

Then G has an ear decomposition $G_1 \subset G_2 \subset \ldots \subset G_r$ such that G_1 is a bi-subdivision of K_4.

Note that K_4 has exactly four odd faces, and so does G_1.

We conclude that G has at least four odd faces.

Proposition

A K_4-based planar brick must have at least four odd faces.

K_4-based planar bricks

K_4-based planar bricks

Theorem (K. and Murty)

Let G be a planar brick. If G has at least four odd faces then G is K_4-based.

K_4-based planar bricks

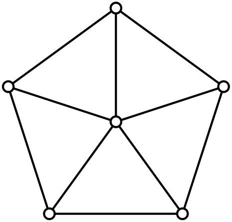
Theorem (K. and Murty)

Let G be a planar brick. If G has at least four odd faces then G is K_4-based.

In other words, the K_4-free planar bricks are precisely those which have exactly two odd faces.

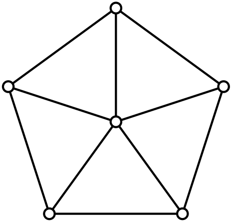
Two infinite families of $\overline{C_6}$-free planar bricks

Two infinite families of $\overline{C_6}$-free planar bricks

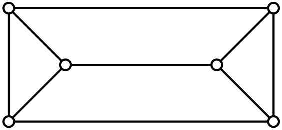


An odd wheel

Two infinite families of $\overline{C_6}$-free planar bricks

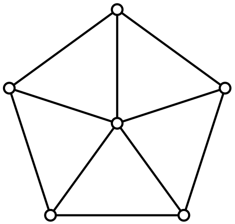


An odd wheel

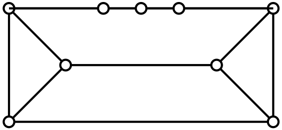


An odd staircase

Two infinite families of $\overline{C_6}$-free planar bricks

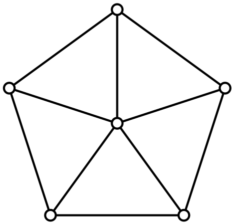


An odd wheel

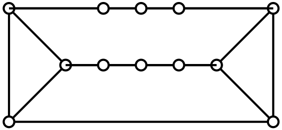


An odd staircase

Two infinite families of $\overline{C_6}$-free planar bricks

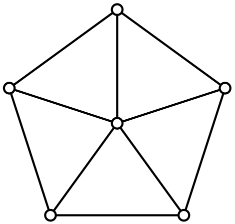


An odd wheel

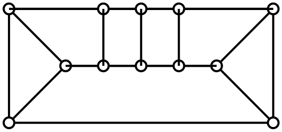


An odd staircase

Two infinite families of $\overline{C_6}$-free planar bricks

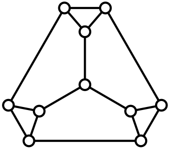


An odd wheel



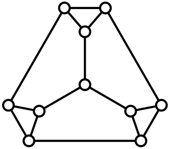
An odd staircase

$\overline{C_6}$-based planar bricks



Tricorn

$\overline{C_6}$-based planar bricks

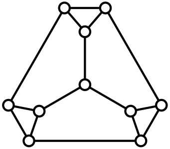


Tricorn

Theorem (K. and Murty)

Let G be a planar brick. If G is **not** *an odd wheel or an odd staircase or the Tricorn, then G is $\overline{C_6}$-based.*

$\overline{C_6}$-based planar bricks



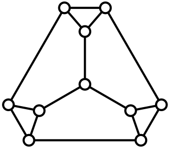
Tricorn

Theorem (K. and Murty)

Let G be a planar brick. If G is **not** *an odd wheel or an odd staircase or the Tricorn, then G is $\overline{C_6}$-based.*

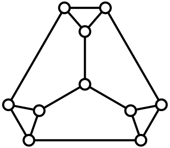
In other words, the $\overline{C_6}$-free planar bricks are precisely the odd wheels, the odd staircases and the Tricorn.

Why tricorn?

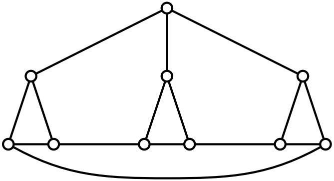


Tricorn

Why tricorn?



Tricorn



Tricorn