Perfect 1-Factorisations of Circulant Graphs of Degree 4

Sarada Herke

PhD Supervisor: Dr. Barbara Maenhaut

The University of Queensland

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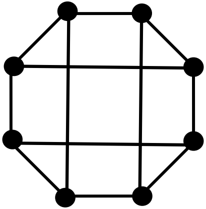
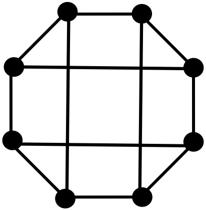
Outline

- definitions and history
- what does bipartite have to do with it?
- our results
- future research

Basic Definitions





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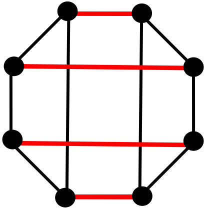

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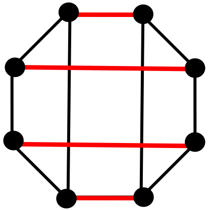

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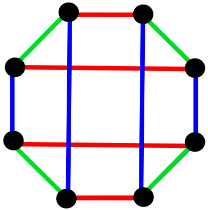

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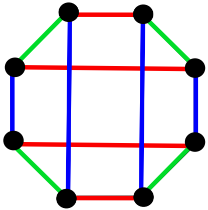

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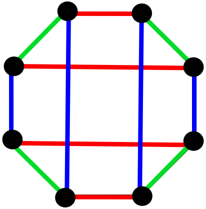

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- The above 1-factorisation is not a P1F.

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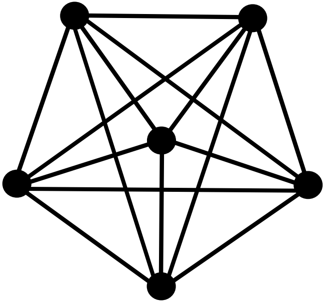
Consider K_6:



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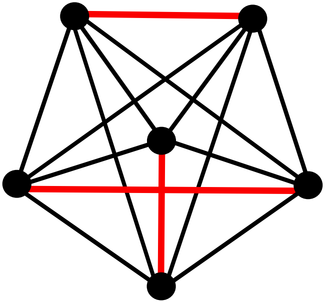

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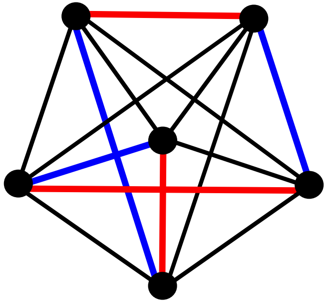

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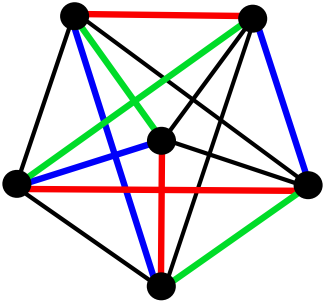

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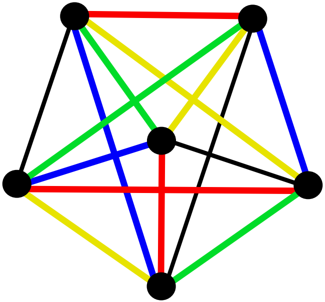

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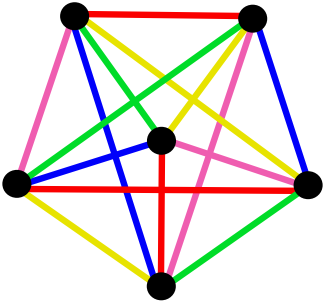

Conjecture for Complete Graphs

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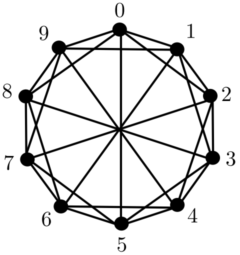
- proven when n is an odd prime
- proven when $2n - 1$ is an odd prime
- small values (upto K_{52}) and other sporadic values

Circulant Graph

Suppose n is even and $S \subseteq \{1, 2, \ldots, \frac{n}{2}\}$.

The circulant graph on n vertices with connection set S, denoted $\mathrm{Circ}(n, S)$, has vertex set $V = \{0, 1, \ldots, n-1\}$ and edge set $E = \{\{x, x + s \pmod{n}\} \mid x \in V, s \in S\}$.

Example: $\mathrm{Circ}(10, \{1, 2, 5\})$



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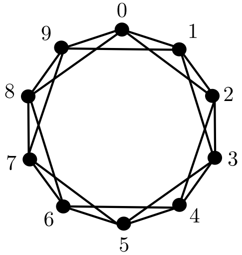

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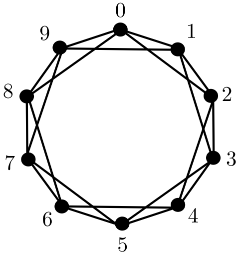

A 4-regular circulant has $S = \{a, b\}$ where $1 \leq a < b < \frac{n}{2}$.

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Problem

Characterise the circulant graphs that admit a P1F.

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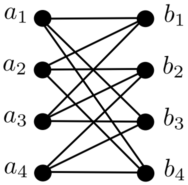

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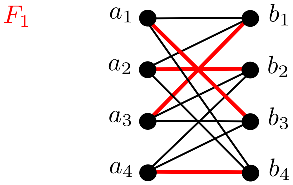

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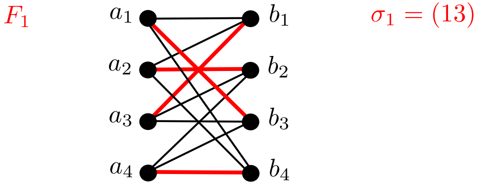

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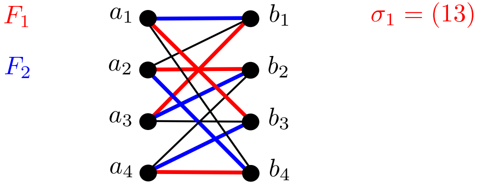

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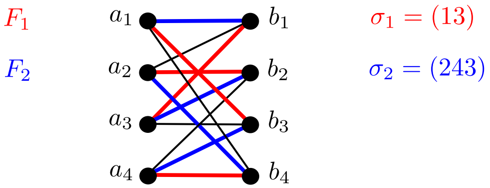

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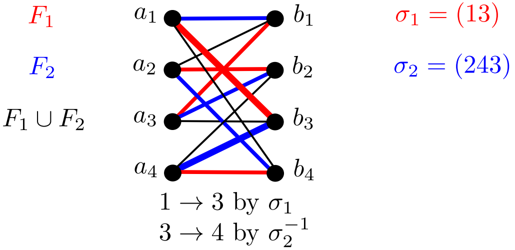

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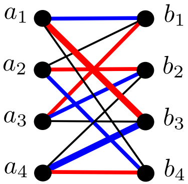

$\sigma_1 = (13)$

$\sigma_2 = (243)$

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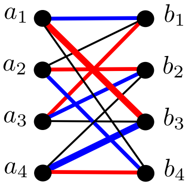

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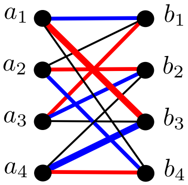

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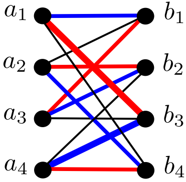

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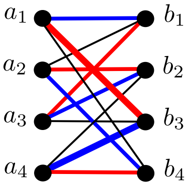

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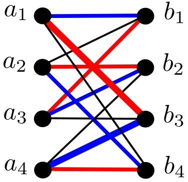

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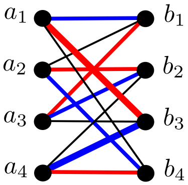

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Theorem (S.H. and Maenhaut)

If $n > 6$, then a connected **3-regular** *circulant graph G of order n admits a P1F if and only if $n \equiv 2 \pmod 4$ and G is bipartite.*

Our Results

Fact (computer results)

For $8 \leq n \leq 28$, *a connected* **4-regular** *circulant* $G = \mathrm{Circ}(n, \{a, b\})$ *has a P1F if and only if* $n \equiv 2 \pmod 4$ *and* G *is bipartite.*

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Problem 3: Why is there no P1F of $\mathrm{Circ}(30, \{1, 11\})$? Are there others like it?

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Then $\mathrm{Circ}(n, \{1, \frac{n}{2} - 1\})$ does not admit a P1F.

Bipartite Constructions

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Bipartite Constructions

n	P1F	unknown	no P1F
30	$\{1,3\}$ $\{1,5\}$ $\{1,7\}$ $\{1,9\}$ $\{3,5\}$	none	$\{1,11\}$
34	$\{1,3\}$ $\{1,5\}$ $\{1,9\}$ $\{1,13\}$	none	none
38	$\{1,3\}$ $\{1,5\}$ $\{1,7\}$ $\{1,9\}$	none	none
42	$\{1,3\}$ $\{1,5\}$ $\{1,11\}$ $\{1,13\}$	$\{1,7\}$ $\{1,9\}$ $\{1,15\}$ $\{3,7\}$	
46	$\{1,3\}$ $\{1,5\}$ $\{1,7\}$ $\{1,11\}$ $\{1,17\}$	none	none
50	$\{1,3\}$ $\{1,7\}$ $\{1,9\}$ $\{1,13\}$	$\{1,5\}$ $\{1,15\}$ $\{1,19\}$	

from $\{1,3\}$ result
from $\{1,b\}$ result
from $\{1,\frac{n}{2}-2\}$ result
from other existence results

WHAT ABOUT $\mathrm{Circ}(30, \{1, 11\})$?

Problem 3: Why is there no P1F of $\mathrm{Circ}(30, \{1, 11\})$? Are there others like it?

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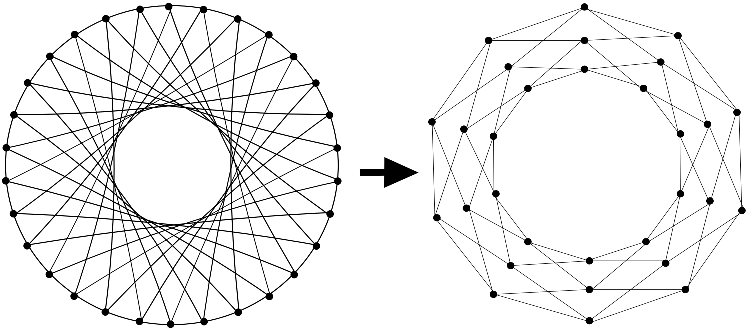

Implications

Theorem (S.H. and Maenhaut)

Suppose $k \equiv 2 \pmod 4$ *and* $k > 6$.
If $k \equiv 10 \pmod{12}$ *then* $\mathrm{Circ}(3k, \{1, k+1\})$ **does not** *admit a P1F.*

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$k \equiv 10 \pmod{12}$	**no P1F**	$k \equiv 2 \pmod{12}$	P1F
10	$\mathrm{Circ}(30, \{1, 11\})$	14	$\mathrm{Circ}(42, \{1, 13\})$
22	$\mathrm{Circ}(66, \{1, 23\})$	26	$\mathrm{Circ}(78, \{1, 25\})$
34	$\mathrm{Circ}(102, \{1, 35\})$	38	$\mathrm{Circ}(114, \{1, 37\})$
46	$\mathrm{Circ}(138, \{1, 47\})$	50	$\mathrm{Circ}(150, \{1, 49\})$

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Corollary (S.H. and Maenhaut)

There is an infinite family of 4-regular bipartite circulant graphs of order $n \equiv 2 \pmod 4$ *that do not admit a P1F.*

Implications

By studying similar structures...

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Theorem (S.H. and Maenhaut)

If $k \equiv 22, 34, 46, 58 \pmod{60}$ *then there exists a P1F of* $\mathrm{Circ}(5k, \{1, b\})$, *where* $b = k-1, 2k+1, 2k-1, k+1$, *respectively.*

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k	existence of P1F	k	existence of P1F
22	$\mathrm{Circ}(110, \{1, 21\})$	82	$\mathrm{Circ}(410, \{1, 81\})$
34	$\mathrm{Circ}(170, \{1, 69\})$	94	$\mathrm{Circ}(470, \{1, 189\})$
46	$\mathrm{Circ}(230, \{1, 91\})$	106	$\mathrm{Circ}(530, \{1, 211\})$
58	$\mathrm{Circ}(290, \{1, 59\})$	118	$\mathrm{Circ}(590, \{1, 119\})$

Future Research

Open Problem

Characterise the bipartite 4-regular circulants of order $2 \pmod 4$ *that admit a P1F.*

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Conjecture

A non-bipartite 4-regular circulant of order at least 8 does not admit a P1F.

Thank you!

Any questions?