
REINHARD DIESTEL, Hamburg University

From pretty pictures to infinite matroids via graph homology: a surprising connection

It has recently (2004–11) been shown that the first homology of locally finite infinite graphs G , such as Cayley graphs of finitely generated groups, is best captured by their so-called ‘topological cycle space’, an \mathbb{F}_2 vector space generated by possibly infinite sums from the edge sets of topological circles in the end-compactification of G .

Solving a 1966 problem of Rado, we recently (2010) proved that infinite matroids with duality can be axiomatized much like finite matroids. This opens up new possibilities in infinite matroid theory that had so far been precluded by the lack of duality, a concept central to finite matroid theory. All infinite matroids now have duals, including such basic finitary matroids as vector spaces of any dimension, or the finite-bond and cycle matroids of an infinite graph.

In studying those duals, we came across a tantalizing connection between graphic matroids and the graph homology outlined above: a connection that points to some hidden theory underlying both, which is still largely mysterious.