

---

## Probabilistic Combinatorics

(Organizer and Chair / Responsable et président: **Mike Molloy** (University of Toronto))

---

---

**TOM BOHMAN**, Carnegie Mellon

*Self-correcting estimates for the triangle free process*

The triangle-free process begins with an empty graph on  $n$  vertices and iteratively adds edges chosen uniformly at random subject to the constraint that no triangle is formed. In this talk we discuss how the graph produced by  $i$  steps of the triangle-free process resembles a graph chosen uniformly at random from the collection of all graphs on  $n$  vertices with  $i$  edges. As a Corollary we get an improved lower bound on the Ramsey number  $R(3, t)$ .

Joint work with Peter Keevash. Similar results were obtained simultaneously and independently by Fiz Pontiveros, Griffiths and Morris.

---

**AMIN COJA-OGHLAN**, Goethe University

*Chasing the  $k$ -SAT threshold*

Let  $F$  be a random Boolean formula in conjunctive normal form over  $n$  Boolean variables with  $m$  clauses of length  $k$ . The existence of a (non-uniform) sharp threshold for the satisfiability of such formulas is well known [Friedgut 1999]. However, despite considerable effort the precise location of this phase transition remains unknown for any  $k > 2$ . The best previous upper and lower bounds differ by an additive  $k \ln 2/2$  [Achlioptas, Peres 2003]. In this talk I present an improved lower bound, which reduces the gap to  $0.19$ . The proof is inspired by the cavity method of statistical mechanics.

---

**DAVID GALVIN**, University of Notre Dame

*Colouring regular bipartite graphs, cubes and grids*

What can we say about a uniform  $q$ -colouring of a regular bipartite graph? Quite a bit, it transpires, despite the question's vagueness. For example, if  $q$  is even then with high probability all colour classes are (essentially) the same size.

Given more structure, we can say more. For example, we have a precise description of the colour classes in a random colouring of the hypercube.

Beyond the hypercube is the integer lattice, and a decades old conjecture of Kotecky that has only recently seen progress.

In part joint work with J. Engbers, J. Kahn, D. Randall and G. Sorkin

---

**MIKE MOLLOY**, University of Toronto

*Clusters of solutions to random linear equations*

Strong evidence from statistical physics indicates that for many models of random constraint satisfaction problems (eg. random  $k$ -SAT, random graph colouring) the set of solutions partitions into *clusters*. We can move throughout the solutions of a cluster by making small local changes, but moving to another cluster requires a large global change.

Clustering is best understood, rigorously, for a random system of boolean linear equations, each on exactly  $k$  variables. We discuss this model, including when the density is within the clustering window.

This includes joint work with Dimitris Achlioptas and with Jane Gao.

---

**BRUCE REED**, McGill University

*Variants of the Erdos-Sos Conjecture*

The Erdos-Sos Conjecture states that if a graph has average degree exceeding  $t-2$  then it contains every tree with  $t$  vertices as a subgraph. This result was proved by Simonovitz and Szemerédi for large  $t$ . We discuss some variants of the conjecture which replace average degree with minimum degree and/or replace subgraph with minor.