## Algebraic Combinatorics

(Organizer and Chair / Responsable et président: **Steph Van Willigenburg** (University of British Columbia))

**DREW ARMSTRONG**, University of Miami

*Maximal Chains of Parabolic Subgroups*

Erdös, Guy and Moon (1973) observed that the number of maximal chains of partitions of the set $\{1,2,\dots,n\}$ is equal to $n!(n-1)!/2^{n-1}$. The proof is easy and it is based on this fact:

$$\binom{n}{2}\binom{n-1}{2}\cdots\binom{2}{2}=\frac{n!(n-1)!}{2^{n-1}}.$$

I will generalize this result to a finite Coxeter group $W$ of rank $r$. Specifically, I will show that the number of maximal chains of parabolic subgroups of $W$ is equal to $r!|W|/2^r$. (Taking the symmetric group $W=S_n$ with rank $r=n-1$ recovers the classical result.) The same method can be used to count chambers of Shi hyperplane arrangements by the dimension of their recession cone.

**ABRAHAM BROER**, Université de Montréal

*Algorithms of making linebundles on cotangent bundles of complete homogeneous spaces more positive.*

Let $X$ be the cotangent bundle of a complete homogeneous space of a complex reductive group $G$ and $L$ a linebundle on $X$. Any cohomology group $H^i(X,L)$ has the structure of a graded $AG$-module, where $A$ is the ring of global regular functions on $X$.

We have developed methods to modify pairs $(X,L)$ to similar pairs $(X',L')$ without changing $A$ and without changing some other property like the Euler characteristic (respectively $H^0$, or without changing any $H^i$) but where $L'$ is strictly more positive than $L$. Modification depends on the property to be preserved.

Joint work with Ascah-Coallier and Jauffret.

**ROSA ORELLANA**, Dartmouth College

*The quasi-partition algebra*

We introduce the quasi-partition algebra $QP_k(n)$ as a centralizer algebra of the symmetric group. We construct a basis for $QP_k(n)$, give a formula for its dimension in terms of the Bell numbers, and describe a set of generators. In addition, we study its representation theory, giving dimensions and indexing sets of irreducible representations. We also provide the Bratteli diagram for the tower of quasi-partition algebras (constructed by letting $k$ range over the positive integers).

Joint work with Z. Daugherty

**BRENDON RHOADES**, University of California, San Diego

*Extending the parking space*

A length $n$ sequence $(a_1,\dots,a_n)$ of positive integers is called a *parking function of size* $n$ if its nondecreasing rearrangement $(b_1\leq\cdots\leq b_n)$ satisfies $b_i\leq i$ for all $i$. We prove that the action of the symmetric group $S_n$ on the vector space spanned by size $n$ parking functions extends to an action of $S_{n+1}$. We construct this extended module explicitly as a graded $S_{n+1}$ representation and describe its restricted graded Frobenius character. This is joint with Andrew Berget at the University of Washington.

**HUGH THOMAS**, University of New Brunswick
*A reflection group perspective on c-vectors*

$c$-vectors are part of the framework of a cluster algebra. Though late-comers in cluster algebra theory, they play a powerful role: it was shown by Nakanishi and Zelevinsky that many facts about cluster algebras follow by an elementary argument once it is known that each $c$-vector has all entries either non-negative or non-positive. I will give a classification of the collections of vectors that can appear as the $c$-vector of a cluster in a skew-symmetrizable cluster algebra starting from an acyclic seed, in terms of reflection groups. This is joint work with David Speyer, arXiv:1203.0277.