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Maximal Chains of Parabolic Subgroups

Erdös, Guy and Moon (1973) observed that the number of maximal chains of partitions of the set $\{1, 2, \ldots, n\}$ is equal to $n!(n-1)!/2^{n-1}$. The proof is easy and it is based on this fact:

$$\binom{n}{2}\binom{n-1}{2}\cdots\binom{2}{2} = \frac{n!(n-1)!}{2^{n-1}}.$$

I will generalize this result to a finite Coxeter group W of rank r. Specifically, I will show that the number of maximal chains of parabolic subgroups of W is equal to $r!|W|/2^r$. (Taking the symmetric group $W = S_n$ with rank $r = n-1$ recovers the classical result.) The same method can be used to count chambers of Shi hyperplane arrangements by the dimension of their recession cone.