Embeddings and Geometric Representations of Graphs

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The edge-ratio of geometric embeddings

A geometric embedding of a planar graph represents vertices by points and edges by non-crossing straight-line segments. We consider the ratio between the length of the longest edge and the shortest edge, more precisely, for a given graph we consider the infimum of this ratio over all geometric embeddings. While the ratio is unbounded in general, it is bounded for the classes of outer-planar graphs and bipartite graphs.

ANDREW BEVERIDGE, Macalester College

Directed Visibility Number for Planar Digraphs and Tournaments

The directed visibility number $b(G)$ of a digraph G is the minimum k such that each vertex of G can be represented by at most k horizontal segments in $\mathbb{R}^2$ with each arc (u, v) of G is realized by an upward visibility from a u-segment to a v-segment. We show that directed planar digraphs have $b(G) \leq 4$, and outerplanar digraphs have $b(G) \leq 3$. Also, every n-tournament has $b(G) \leq (n+1)/3 + 3$ and large transitive tournaments satisfy $b(G) \leq (3n + 11)/14 + 41$.

SALLY COCKBURN, Hamilton College

Permutations and Geometric Realizations of $K_{2,n}$

Two geometric realizations of G are geo-isomorphic if there is an isomorphism between them preserving edge crossings and non-crossings. Geo-homomorphisms, an extension of graph homomorphisms, define a partial order on geo-isomorphism classes. We investigate the homomorphism poset of $K_{2,n}$ by establishing a correspondence between realizations of $K_{2,n}$ and S_n, in which edge crossings correspond to inversions. We provide the number of geo-isomorphism classes for $n \leq 9$ and the complete poset for $n \leq 5$.

ALICE DEAN, Skidmore College

Posets of Geometric Graphs

Joint work with Debra Boutin, Sally Cockburn, and Andrei Margea.

We use geometric homomorphisms to introduce a partial order on the isomorphism classes of geometric realizations of a graph G. We develop tools to determine when two geometric realizations are comparable and give results for P_n, C_n and K_n.

MARK ELLINGHAM, Vanderbilt University

Hamilton cycle embeddings of complete tripartite graphs

Ellingham and Stephens showed that if $m \geq n - 1$ then in all but one case the nonorientable genus of $\overline{K_m} + K_n$ is the same as that of its subgraph $K_{m,n}$. Only partial results are known for the orientable case. We give new results using hamilton cycle embeddings of complete tripartite graphs $K_{n,n,n}$, which are of independent interest. We construct these embeddings using a variety of techniques. This is joint work with Justin Z. Schroeder.