Cycle-continuous mappings – order structure

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Outline

1. Introduction

2. Our results
   - Snarks
   - Tree of snarks

3. Future work
Nice open problem

Problem (Cycle Double Cover [Seymour, Szekeres, Tutte?])

For every bridgeless graph $G$ exists a list cycles $C_1, \ldots, C_k$ such that every edge of $G$ is in exactly two of them.

- possibly we may take $k = 5$ [Celmins, Preissmann]
- *cycle* = even graph = set of edges with all degrees even
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An approach

Definition (Jaeger 1980/DeVos, Nešetřil, Raspaud 2006)

\( G, H \) \ldots graphs
\( f : E(G) \rightarrow E(H) \) \ldots mapping
\( f \) is cycle-continuous (cc) iff for every cycle \( C \) in \( H \) the preimage \( f^{-1}(C) \) is a cycle in \( G \). The existence of some cycle-continuous mapping from \( G \) to \( H \) is denoted by \( G \xrightarrow{cc} H \).

- For a cubic graph \( G \xrightarrow{cc} K_2^3 \) iff \( G \) admits a 3-edge-coloring
- Let a mapping \( f : E(G) \rightarrow E(H) \) be such that for each vertex \( v \) of \( G \), it maps all edges incident with \( v \) to all edges incident with some vertex of \( H \). Then \( f \) is cycle-continuous.
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- For a cubic graph $G \xrightarrow{cc} K_3^2$ iff $G$ admits a 3-edge-coloring
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\[ f \text{ is } \text{cycle-continuous} \ (cc) \text{ iff for every cycle } C \text{ in } H \text{ the preimage } f^{-1}(C) \text{ is a cycle in } G. \] The existence of some cycle-continuous mapping from $G$ to $H$ is denoted by $G \xrightarrow{cc} H$.

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The hope

Conjecture (Jaeger 1981)

Every bridgeless graph has a cycle continuous mapping to the Petersen graph.

- true for all 3-edge-colorable graphs
- true for all graphs up to 36 vertices
- and many others (e.g., for all Flower snarks)
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Related question

**Question (DeVos, Nešetřil, Raspaud 2006)**

*Is there an infinite family of bridgeless graphs such that there is no cycle-continuous mapping between any two of them?*

- [DNR] there is an arbitrarily large finite such family
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Every countable poset can be represented by a family of bridgeless graphs and existence of cycle-continuous mapping between them.
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Plan of the proof

- In general, the cycle-continuous mapping behaves very erratically.
- We tame it by using specially crafted graphs to make it behave like homomorphisms.
- Critical snarks are crucial for this.
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Snarks

From now on we deal with cubic graphs only.

- bridgeless graph $G$ is a **snark** if it is not 3-edge-colorable; equivalently, if $G \not\cc \rightarrow K_2^3$
- a snark $G$ is **critical** if for every edge $e$ we have $G - e \cc \rightarrow K_3^3$; equivalently, $G - e$ is not 3-edge-colorable ($\overline{H}$ denoting $H$ with suppressed vertices of degree 2) [DeVos, Nešetřil, Raspaud; da Silva, Lucchesi; Nedela, Škoviera]
- example: Petersen graph, Blanuša snarks on 18 vertices

**Theorem (DNR 2006)**

Suppose $G$, $H$ are critical snarks, cyclically 4-edge-connected, $|E(G)| = |E(H)|$.
Then $G \cc \rightarrow H$ iff $G \cong H$.
Moreover, all cycle-continuous mappings from $G$ to $H$ are induced by the isomorphism.
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Moreover, all cycle-continuous mappings from $G$ to $H$ are induced by the isomorphism.
Critical snarks

Lemma

There are two snarks $B_1, B_2$ with 18 vertices, that are critical and nonisomorphic. Moreover, none of $B_1, B_2$ is vertex transitive; in particular, there is no isomorphism $f : V(B_2) \rightarrow V(B_2)$ for which $f(a) = b$. 

![Diagram of two snarks with 18 vertices](image)
Snark constructions: 3-join

Lemma

For any graphs $G_1, G_2$ we have $G_i \xrightarrow{cc} G_1 \oplus_3 G_2$ for $i = 1, 2$.

Lemma

Let $G_1, G_2$ be graphs.
Let $K$ be a cyclically 4-edge-connected cubic graph that is 2-transitive.
Then $G_1 \oplus_3 G_2 \xrightarrow{cc} K$ if and only if $G_1 \xrightarrow{cc} K$ and $G_2 \xrightarrow{cc} K$. 
Snark constructions: 3-join

\[
\begin{align*}
\bigoplus_3 G_1 & \rightarrow G_1 \oplus_3 G_2 \\
\end{align*}
\]

\[
\begin{align*}
\Leftrightarrow & \\
\end{align*}
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Snark constructions: 3-join

\[ \oplus_3 \]

\[ \equiv \]

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Lemma

Let $G_1, G_2$ be graphs. Let $K$ be a cyclically 4-edge-connected cubic graph that is 2-transitive. Then $G_1 \oplus_3 G_2 \stackrel{cc}{\longrightarrow} K$ if and only if $G_1 \stackrel{cc}{\longrightarrow} K$ and $G_2 \stackrel{cc}{\longrightarrow} K$.

Corollary

Let $G_1, G_2$ be cubic bridgeless graphs. Then $G_1 \oplus_3 G_2$ is a snark, iff at least one of $G_1, G_2$ is a snark.

Corollary

Let $G_1, G_2$ be cubic bridgeless graphs. If $G_1 \oplus_3 G_2 \not\stackrel{cc}{\longrightarrow} \text{Pt}$ then $G_i \not\stackrel{cc}{\longrightarrow} \text{Pt}$ for some $i \in \{1, 2\}$. 
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Let $G_1, G_2$ be cubic bridgeless graphs. If $G_1 \oplus_3 G_2 \not\xrightarrow{cc} Pt$ then $G_i \not\xrightarrow{cc} Pt$ for some $i \in \{1, 2\}$.
Introduction

Our results
- Snarks
- Tree of snarks

Future work
Tree of snarks – construction

$G_1$ $G_2$ $G_3$
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$G_1$  $G_2$  $G_3$
Tree of snarks – construction
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$G_1$ $G_2$ $G_3$
Tree of snarks – construction

\[ G_1 \quad G_2 \quad G_3 \]
Tree of snarks – construction

$$\mathcal{G} = \{G_1, G_2, G_3, \ldots\}$$

$$\mathcal{T}(\mathcal{G})$$

Critical snarks
Tree of snarks – properties

Lemma

\( \mathcal{G} \) – critical snarks, cyclically 4-edge-connected, pairwise nonisomorphic, all of the same size

\( H \in T(\mathcal{G}) \) and \( K \in \mathcal{G} \)

Then

\[ K \overset{\text{cc}}{\longrightarrow} H \iff K \cong G_i \text{ for some } G_i \in \mathcal{G} \text{ s.t. } i \text{ used on } T \]

Moreover, all mappings \( K \overset{\text{cc}}{\longrightarrow} H \) are an isomorphism on \( K \) composed with \( \iota_v \) for some \( v \in V(G) \) for which \( c(v) = i \).
Tree of snarks – properties

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Tree of snarks – properties

$G_1$ $G_2$ $G_3$
Tree of snarks – properties

$T(G)$

$G_4$
Tree of snarks – properties

\[ T(G) \]

\[ G_4 \]
Tree of snarks – properties

**Theorem**

\[ T_1, T_2 \ldots \text{trees} \]
\[ c_i : V(T_i) \to [n], H_i \in T_i(G) \ (i = 1, 2) \]

Every \( g : H_1 \xrightarrow{\text{cc}} H_2 \) is guided by a homomorphism \( f : T_1 \to T_2 \) of reflexive colored graphs: \( \exists f : V(T_1) \to V(T_2) \) such that

- \( c_2(f(v)) = c_1(v) \) \( f \) respects colors, and
- if \( uv \) is an edge of \( T_1 \), then \( f(u)f(v) \) is an edge of \( T_2 \) or \( f(u) = f(v) \).

Moreover, \( g \) induces a cc mapping on the blocks.
Theorem

$T_1, T_2 \ldots$ trees

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Moreover, $g$ induces a cc mapping on the blocks
Tree of snarks – properties
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Infinite antichain

**Theorem (Š. 2012)**

*There is an infinite family of bridgeless graphs such that there is no cycle-continuous mapping between any two of them.*

**Proof:**

- \( \mathcal{G} = \{B_1, B_2\} \), fix \( a, b \in V(B_2) \) so that no automorphism maps \( a \mapsto b \)
- \( T_n = \) a path colored as \( 1(2)^{n-1}1 \)
- \( G_n \in T_n(\mathcal{G}) \), taking always “\( a \) on the left, \( b \) on the right”
- Due to the choice of \( a, b \), the “folding” in the previous figure is not possible.
- Thus, \( \{G_n, n \in \mathbb{N}\} \) is an antichain.
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Proof:

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- Due to the choice of $a, b$, the “folding” in the previous figure is not possible.
- Thus, $\{ G_n, n \in \mathbb{N} \}$ is an antichain.
Another ingredient

Theorem (Hubička, Nešetřil 2005)

*Arbitrary countable poset can be represented by finite directed paths and existence of homomorphisms between them.*
Universal poset

Theorem (Š. 2012)

Every countable poset can be represented by a family of cubic bridgeless graphs and existence of cycle-continuous mapping between them.
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- Using Hubička and Nešetřil, we need to find $m : \{\text{dir. paths}\} \rightarrow \{\text{cubic bridgeless graphs}\}$
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- $\mathcal{G} = \{B_2\}$, fix $a, b \in V(B_2)$ so that no automorphism maps $a \mapsto b$
Open problems

- Is there an infinite antichain in cc mappings using cubic, cyclically 4-edge-connected graphs?
- Are there gaps in the poset of cc mappings? I.e., are there $G, H$ s.t. $G \xrightarrow{cc} H$ but for no $K$ we have $G \xrightarrow{cc} K \xrightarrow{cc} H$ unless $K \xrightarrow{cc} G$ or $H \xrightarrow{cc} K$?
- $K_2^3 \xrightarrow{cc} \text{Pt}$ is not a gap – for example $K_2^3 \prec cc B_1 \prec cc \text{Pt}$
- Not to forget the original question: $G$ cubic bridgeless $\Rightarrow G \xrightarrow{cc} \text{Pt}$?
- If $G$ is a minimal counterexample, can $G$ contain a 4-cycle?
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