Unique graph representations of bias matroids

Daryl Funk

joint with

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CanaDAM
Memorial University of Newfoundland
June 11, 2013
Circuits in graphs

- A circuit is the edge set of a cycle

<table>
<thead>
<tr>
<th>Graph</th>
<th>Circuits</th>
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<tbody>
<tr>
<td>{a, b, c, d}</td>
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<tr>
<td>{a, b, c, e, f, h}</td>
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Circuits in graphs

- A circuit is the edge set of a cycle

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Circuits in graphs

- A *circuit* is the edge set of a cycle

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Question: When do two graphs have the same set of circuits?
When do two graphs have the same set of circuits?

- splitting cut vertices,
- gluing on a vertex does not change the circuits
When do two graphs have the same set of circuits?

- flipping on a 2-separation
When do two graphs have the same set of circuits?

- flipping on a 2-separation does not change the circuits
When do two graphs have the same set of circuits?

Theorem (Whitney, 1933)

Two graphs $G$ and $H$ have the same set of circuits if and only if $G$ and $H$ are isomorphic up to

- splitting or gluing cut vertices
- Whitney flips on a 2-separation
When do two biased graphs have the same set of circuits?

Theorem (DeVos, F., Goddyn, Pivotto)

When $G$ is 3-connected, the circuits of $(G, B)$ are uniquely determined by $(G, B)$, unless $(G, B)$ is in one of a few exceptional families.
Circuits in graphs

Given a collection of subsets $C$ of a set $E$, is there a graph with these as its circuits? If so, call $(E, C)$ graphic.

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# Circuits in graphs

\[ E = \{ a, b, c, d \} \]

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Circuits in graphs

\[ E = \{a, b, c, d\} \]
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<td>![Graph with vertices a, b, c, d and edges a-b, b-c, c-d]</td>
<td>![Red X]</td>
</tr>
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- \{a, b, c\}
- \{a, b, d\}
- \{a, c, d\}
- \{b, c, d\}
**Question**: When is a collection of subsets the set of circuits of a graph?

**Theorem (Tutte, 1959)**

A collection $C$ of subsets of a ground set $E$ is graphic if and only if

- $C$ satisfies three certain properties and
- $C$ does not have any one of five certain minors.
Three properties of circuits

- no circuit is properly contained in another
Three properties of circuits

- no circuit is properly contained in another
Three properties of circuits

- if \( e \in C_1 \cap C_2 \), then there is another circuit contained in \( (C_1 \cup C_2) - e \)
Three properties of circuits

- if $e \in C_1 \cap C_2$, then there is another circuit contained in $(C_1 \cup C_2) - e$
Three properties of circuits

- if $e \in C_1 \cap C_2$, then there is another circuit contained in $(C_1 \cup C_2) - e$
Three properties of circuits

- the circuit elimination axiom
Three properties of circuits

- $\emptyset$ is not a circuit
Three properties of circuits

- $\emptyset$ is not a circuit
- no circuit is properly contained in another
- the *circuit elimination axiom*
Abstracting circuits: Matroids

Let $E = \{e_1, e_2, \ldots, e_m\}$ be a set

Let $C$ be a collection of subsets of $E$, called circuits, such that

- $\emptyset$ is not a circuit
- no circuit is properly contained in another
- the circuit elimination axiom holds

Such a pair $(E, C)$ is called a matroid
Example: Uniform matroids

\[ U_{r,n} \]

- ground set \{1, \ldots, n\}
- circuits are all subsets of size \( r + 1 \)
Example: Uniform matroids

\( U_{r,n} \)
- ground set \( \{1, \ldots, n\} \)
- circuits are all subsets of size \( r + 1 \)
  - circuit properties hold?
    - \( \emptyset \) not a circuit
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\begin{center}
\includegraphics[width=\textwidth]{example_matroid_diagram.png}
\end{center}
Example: Uniform matroids

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\[ C_1 \supseteq C_2 \]

\( e \)

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Example: Uniform matroids

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Example: Uniform matroids

$U_{r,n}$

- ground set $\{1, \ldots, n\}$
- circuits are all subsets of size $r + 1$

- $U_{1,n}$ is graphic

$n = 6$
Example: Uniform matroids

$U_{r,n}$

- ground set $\{1, \ldots, n\}$
- circuits are all subsets of size $r + 1$

- $U_{2,3}$ is graphic
Example: a non-graphic matroid

$U_{r,n}$

- ground set \{1, \ldots, n\}
- circuits are all subsets of size $r + 1$

- $U_{2,4}$ is not graphic
Example: a non-graphic matroid

\[ U_{r,n} \]
- ground set \(\{1, \ldots, n\}\)
- circuits are all subsets of size \(r + 1\)

- \(U_{2,4}\) is not graphic

Put \(E = \{a, b, c, d\}\)

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Example: a non-graphic matroid

$U_{r,n}$
- ground set $\{1, \ldots, n\}$
- circuits are all subsets of size $r + 1$

- $U_{2,4}$ is not graphic

Put $E = \{a, b, c, d\}$
When is $C$ the set of circuits of a graph?

Theorem (Tutte)

A matroid $(E, C)$ is graphic if and only if it does not have any one of five certain minors.
When is $C$ the set of circuits of a graph?

**Theorem (Tutte)**

A matroid $(E, C)$ is graphic if and only if it does not have any one of five certain minors.

- a *minor* of $(E, C)$ is any matroid obtained by a sequence of deletions or contractions
When is $C$ the set of circuits of a graph?

- **deletion** and **contraction** of an element from the ground set of a matroid $(E, C)$ is defined just as for graphs.

\[ G \xrightarrow{e} G \setminus e \]
When is \( C \) the set of circuits of a graph?

- deletion and contraction of an element from the ground set of a matroid \( (E, C) \) is defined just as for graphs.
When is $C$ the set of circuits of a graph?

An excluded minor theorem characterises a family of matroids by exhibiting a list

- Matroid $\in$ Family $\iff$ no minor in list
- members of the list are the excluded minors for the family

Theorem (Wagner/Kuratowski)
A graph can be embedded in the plane if and only if it has no $K_5$ or $K_{3,3}$ as a minor.
When is $C$ the set of circuits of a graph?

An *excluded minor theorem* characterises a family of matroids by exhibiting a list

- Matroid $\in$ Family $\iff$ no minor in list

- members of the list are the *excluded minors* for the family

**Theorem (Wagner/Kuratowski)**

*A graph can be embedded in the plane if and only if it has no $K_5$ or $K_{3,3}$ as a minor.*
When is $C$ the set of circuits of a graph?

Theorem (Tutte)

A matroid $(E, C)$ is graphic if and only if it has no $U_{2,4}$, $F_7$, $F_7^*$, $M^*(K_5)$, or $M^*(K_{3,3})$ minor.
Biased graphs

A biased graph is a pair \((G, B)\)

- a graph \(G\)
- together with a collection of distinguished cycles \(B\)
  - called balanced
Biased graphs

A biased graph is a pair \((G, \mathcal{B})\)

- a graph \(G\)
- together with a collection of distinguished cycles \(\mathcal{B}\)
  - called balanced
  - obeying the theta property

\begin{center}
\begin{tikzpicture}
    \foreach \x in {0,360/10,...,360}
    \draw (\x:1.5cm) -- (\x + 180:1.5cm);
    \foreach \x in {0,360/10,...,360}
    \draw (\x:1.25cm) -- (\x + 360/5:1.25cm);
\end{tikzpicture}
\end{center}

cannot contain exactly two balanced cycles
Example: Graphs on surfaces

Given a graph embedded on a surface

- put $\mathcal{B} = \{\text{contractible cycles}\}$
Example: Signed graphs

Given a graph

- label each edge with $+1$ or $-1$
- put $\mathcal{B} = \{\text{cycles with even } \# \text{ of edges labelled } -1\}$
Example: Signed graphs

Given a graph

- label each edge with +1 or −1
- put $\mathcal{B} = \{\text{cycles with even \# of edges labelled } -1\}$

- giving every edge label $-1$ we get $\mathcal{B} = \{\text{even cycles}\}$
Graphs are biased graphs

- put $\mathcal{B} = \{ \text{all cycles} \}$
Circuits in biased graphs

Given biased graph \((G, \mathcal{B})\), let \(C\) consist of

- balanced cycles
- every subdivision of one of the following, in which all cycles are unbalanced

- tight handcuffs
- loose handcuffs
- odd theta

- this choice for \(C\) satisfies our circuit properties
  - so \((E, C)\) is a matroid
Frame Matroids

Given biased graph \((G, \mathcal{B})\), let \(C\) consist of

- balanced cycles
- every subdivision of one of the following, in which all cycles are unbalanced
  - tight handcuffs
  - loose handcuffs
  - odd theta

- this choice for \(C\) satisfies our circuit properties
  - \((E, C) = M(G, \mathcal{B})\) is a bias or frame matroid
$U_{2,4}$ is frame

\[ a \rightarrow b \rightarrow c \rightarrow d \]

\[ C = \{abc, abd, acd, bcd\} \]
$U_{2,4}$ is frame

- there are three biased graphs whose circuits are the circuits of $U_{2,4}$

\[ C = \{abc, abd, acd, bcd\} \]

- all cycles unbalanced
\( U_{2,4} \) is frame

\[
\begin{align*}
\mathcal{C} &= \{abc, abd, acd, bcd\} \\
\text{• there are three biased graphs whose circuits are the circuits of} \\
U_{2,4}
\end{align*}
\]

- all cycles unbalanced

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SFU
$U_{2,4}$ is frame

- tight handcuffs
- loose handcuffs
- odd theta

\[ C = \{abc, abd, acd, bcd\} \]

- there are three biased graphs whose circuits are the circuits of $U_{2,4}$

- all cycles unbalanced

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When do two biased graphs have the same collection of circuits?
When do two biased graphs have the same collection of circuits?

Theorem (DeVos, F., Goddyn, Pivotto)

When $G$ is 3-connected, $M(G, \mathcal{B})$ are uniquely determined by $(G, \mathcal{B})$, unless $(G, \mathcal{B})$ is in one of a few exceptional families.
When do two biased graphs have the same collection of circuits?

Theorem (DeVos, F., Goddyn, Pivotto)

When $G$ is 3-connected, $M(G, B)$ are uniquely determined by $(G, B)$, unless $(G, B)$ is in one of a few exceptional families.

- When can the circuits of a biased graph be realised as the set of circuits of a graph?
Which frame matroids are graphic?

- i.e., which biased graphs have circuits = the circuits of a graph?
When can the circuits of a biased graph be realised as the set of circuits of a graph?

Theorem

A connected frame matroid \((E, C) = M(G, B)\) is graphic if and only if \((G, B)\) is in one of five families of biased graphs.

Another characterization has been obtained by Zaslavsky [1987] and Slilaty [2006], in terms of 1,2,3-sums of balanced and projective planar signed graphs.
Which biased graphs have circuits = the circuits of a graph?

When \((G, B)\) is balanced

- i.e. \(B = \{\text{all cycles}\}\)
- i.e. \((G, B)\) is a graph
Which biased graphs have circuits = the circuits of a graph?

When \((G, B)\) is a \textit{pinch}
Which biased graphs have circuits = the circuits of a graph?

When \((G, \mathcal{B})\) is a pinch

\[
\begin{array}{c|c}
\text{graph} & \text{biased graph} \\
\hline
\text{balanced} & \text{handcuffs} & \text{odd theta} \\
\end{array}
\]
Which biased graphs have circuits = the circuits of a graph?

When \((G, B)\) is a pinch

\begin{align*}
\text{graph} & \quad \text{biased graph} \\
\begin{array}{c}
\begin{tikzpicture}
\draw[fill=black] (0,0) circle (0.1cm);
\draw[fill=black] (1,0) circle (0.1cm);
\draw[-,red] (0,0) -- (1,0);
\draw[-,black] (0,0) -- (0,1);
\draw[-,black] (0,0) -- (0,-1);
\draw[-,black] (1,0) -- (1,1);
\draw[-,black] (1,0) -- (1,-1);
\end{tikzpicture}
\end{array}
\end{align*}

\text{balanced} \quad \text{handcuffs} \quad \text{odd theta}
Which biased graphs have circuits = the circuits of a graph?

When \((G, \mathcal{B})\) is a pinch

\[
\begin{align*}
\text{graph} & \quad \approx \quad \text{biased graph} \\
\begin{tikzpicture}[scale=0.8]
\draw[very thick] (-1,0) -- (1,0);
\node at (-1,0) [circle,fill,inner sep=2pt] (u) {};
\node at (1,0) [circle,fill,inner sep=2pt] (v) {};
\end{tikzpicture}
\end{align*}
\]
Which biased graphs have circuits = the circuits of a graph?

When \((G, B)\) is a \textit{bud}

- a signed graph: marked edges are labelled \(-1\)
Which biased graphs have circuits = the circuits of a graph?

When \((G, B)\) is a \textit{bud}

\[
\begin{array}{c|c}
\text{graph} & \text{biased graph} \\
\hline
\text{bal} & \text{bal} \\
\text{un} & \text{un}
\end{array}
\]
Which biased graphs have circuits = the circuits of a graph?

When \((G, \mathcal{B})\) is a \textit{bud}

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\begin{array}{c|c}
\text{graph} & \text{biased graph} \\
\hline
\text{bal} & \text{bal} \\
\end{array}
\]
Which biased graphs have circuits = the circuits of a graph?

When \((G, \mathcal{B})\) is a *bud*

\[
\begin{array}{l}
\text{graph} & \text{biased graph} \\
\hline
\text{bal} & \text{bal} \\
\Rightarrow & \Rightarrow \\
\text{balanced} & \\
\text{handcuffs} & \\
\text{odd theta} &
\end{array}
\]
Which biased graphs have circuits = the circuits of a graph?

When \((G, B)\) a 4-leaf clover [Shih, 1982]
- a signed graph: marked edges are labelled \(-1\)

![Diagram of a 4-leaf clover graph](image-url)
A 4-leaf clover

biased graph

graph

A B
CD
A B C D
x y z
xA yA zA
xB yB zB
xC yC zC
xD yD zD

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SFU
A 4-leaf clover

biased graph

graph

D. Funk
SFU
A 4-leaf clover

biased graph

graph

balanced handcuffs odd theta

D. Funk SFU
A 4-leaf clover

biased graph

graph

D. Funk SFU
A 4-leaf clover

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balanced
handcuffs
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D. Funk SFU
Which biased graphs have circuits = the circuits of a graph?

When \((G, B)\) is a fat theta

- cycles are balanced if and only if contained in a single “lobe”
Which biased graphs have circuits = the circuits of a graph?

When \((G, B)\) is a fat theta

- cycles are balanced if and only if contained in a single “lobe”
Which biased graphs have circuits = the circuits of a graph?

When \((G, \mathcal{B})\) is a fat theta

- cycles are balanced if and only if contained in a single “lobe”
Which biased graphs have circuits = the circuits of a graph?

Theorem
A connected frame matroid \((E, C) = M(G, B)\) is graphic if and only if \((G, B)\) is:

1. balanced,
2. a pinch,
3. a bud,
4. a 4-leaf clover, or
5. a fat theta.
A fat theta is balanced is a bud is a pinch
A fat theta is balanced is a bud is a pinch
A fat theta is balanced is a bud is a pinch
A fat theta is balanced is a bud is a pinch
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balanced
handcuffs
odd theta

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When do two biased graphs have the same set of circuits?

Theorem (DeVos, F., Goddyn, Pivotto)

When $G$ is 3-connected, $M = M(G, B)$ is uniquely represented by $(G, B)$, unless $(G, B)$ is in one of a few exceptional families.

Furthermore, we know the few exceptions, and their few different biased graph representations.
• a matroid \((E, C)\) is *connected* if every pair \(e_1, e_2 \in E\) are contained in some circuit
Definition

If $e \notin X$

then $e \in \text{closure}(X)$
Hyperplanes in graphs

- complements of minimal cuts $S'$ are hyperplanes
- maximal unions of circuits subject to $\text{closure}(X) \neq E$
When do two biased graphs have the same set of circuits?

Theorem (DeVos, F., Goddyn, Pivotto)

When \( G \) is 3-connected, \( M = M(G, \mathcal{B}) \) is uniquely represented by \((G, \mathcal{B})\), unless \((G, \mathcal{B})\) is in one of a few exceptional families.
When do two biased graphs have the same set of circuits?

Theorem (DeVos, F., Goddyn, Pivotto)

When $G$ is 3-connected, $M = M(G, \mathcal{B})$ is uniquely represented by $(G, \mathcal{B})$, unless $(G, \mathcal{B})$ is in one of a few exceptional families.

Strategy

- $M(G, \mathcal{B})$ has $n - 1$ connected, non-graphic hyperplanes

$\implies (G, \mathcal{B})$ uniquely represents $M(G, \mathcal{B})$
When do two biased graphs have the same set of circuits?

Theorem (DeVos, F., Goddyn, Pivotto)

When $G$ is 3-connected, $M = M(G, \mathcal{B})$ is uniquely represented by $(G, \mathcal{B})$, unless $(G, \mathcal{B})$ is in one of a few exceptional families.

Strategy

- $M(G, \mathcal{B})$ has $n - 1$ connected, non-graphic hyperplanes
  \[\implies (G, \mathcal{B})\text{ uniquely represents } M(G, \mathcal{B})\]
- otherwise, there are $u, v \in V(G)$ such that
  - $G - u$ and $G - v$ are each graphic
When do two biased graphs have the same set of circuits?

• Each of $G - u$ and $G - v$ is one of:
  1. balanced,
  2. a pinch,
  3. a bud,
  4. a 4-leaf clover, or
  5. a fat theta.

• for each pair, re-construct $G$
The Bean bag family

- $G - u$ balanced, $G - v$ graphic
The Bean bag family

- $G - u$ balanced, $G - v$ graphic

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The Bean bag family

- $G - u$ balanced, $G - v$ graphic
The Bean bag family

- $G - u$ balanced, $G - v$ graphic
The Bean bag family

- \( G - u \) balanced, \( G - v \) graphic
The Bean bag family

- $G - u$ balanced, $G - v$ graphic
The Bean bag family

- $G - u$ balanced, $G - v$ graphic
The Bat graph family

- If \( G - u \) pinched at \( v \), and \( G - v \) pinched at \( u \)
The Bat graph family

- If $G - u$ pinched at $v$, and $G - v$ pinched at $u$
The Bat graph family

- If $G - u$ pinched at $v$, and $G - v$ pinched at $u$
The Bat graph family

- If $G - u$ pinched at $v$, and $G - v$ pinched at $u$:
The Bat graph family

- If $G - u$ pinched at $v$, and $G - v$ pinched at $u$: 

![Diagram showing the Bat graph family](image)
The Bat graph family

- If $G - u$ pinched at $v$, and $G - v$ pinched at $u$: 

$$
\begin{array}{c}
A \quad B \\
C \quad D
\end{array}
\quad \leftrightarrow 
\begin{array}{c}
A \quad B \\
C \quad D
\end{array}
$$
The Bat graph family

- If \( G - u \) pinched at \( v \), and \( G - v \) pinched at \( u \):
When do two biased graphs have the same set of circuits?

And so on...  
- each of $G - u$ and $G - v$ is one of:  
  1. balanced,  
  2. a pinch,  
  3. a bud,  
  4. a 4-leaf clover, or  
  5. a fat theta.  
- for each pair, re-construct $G$
Open

Can we assume only $M = M(G, B)$ 3-connected?
- i.e., find all biased graph representations of a given 3-connected frame matroid
Can we assume only $M = M(G, B)$ 3-connected?

- *i.e.*, find all biased graph representations of a given 3-connected frame matroid

- each of $G - u$ and $G - v$ is one of:
  1. disconnected
  2. balanced,
  3. a pinch,
  4. a bud,
  5. a 4-leaf clover, or
  6. a fat theta.

- for each pair, re-construct $G$
When is $\mathcal{C}$ the set of circuits of a graph?

- **Answer:** Tutte’s excluded minor characterisation

**Theorem (Tutte)**

*A matroid $(E, \mathcal{C})$ is graphic if and only if it has no $U_{2,4}$, $F_7$, $F_7^*$, $M^*(K_5)$, or $M^*(K_{3,3})$ minor.*
Open

When is $\mathcal{C}$ the set of circuits of a biased graph?

The Theorem we’d like
A matroid $(E, \mathcal{C})$ is frame if and only if it has no minor in the list \{\(N, N', \ldots, N''\ldots\}\).
When is $C$ the set of circuits of a biased graph?

The Theorem we’d like
A matroid $(E, C)$ is frame if and only if it has no minor in the list \{\(N, N', \ldots, N''\ldots\)\}.

- Let $N$ be an excluded minor for the class of frame matroids, let $e, f \in E$
  - consider $N \setminus e$ and $N \setminus f$
  - if we know all biased graphs representing $N \setminus e$ and $N \setminus f$ we should be able to reconstruct $N$