Strongly Regular Graphs with non-trivial automorphisms

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Strongly Regular Graph

Definition

A strongly regular graph $\mathrm{srg}(v, k, \lambda, \mu)$ is a graph with v vertices such that the number of common neighbours of x and y is k, λ, or μ according to whether x and y are equal, adjacent, or non–adjacent, respectively.

The Petersen Graph, SRG$(10, 3, 0, 1)$

$$
\left[\begin{array}{c|ccc|ccc|ccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
\hline
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
\hline
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
\hline
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{array}\right]
$$

The Petersen Graph, SRG$(10, 3, 0, 1)$

$$\left[\begin{array}{c|ccc|ccc|ccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
\hline
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
\hline
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
\hline
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{array}\right]$$

$$B^2 = (k - \mu)I + \mu J + (\lambda - \mu)B.$$

Unknown strongly regular graphs with small parameters

v	k	λ	μ
65	32	15	16
69	20	7	5
75	32	10	16
76	30	8	14
76	35	18	14
85	14	3	2
85	30	11	10
85	42	20	21
88	27	6	9
95	40	12	20
96	35	10	14
96	38	10	18
96	45	24	18
99	14	1	2
99	42	21	15
100	33	8	12

Table: (CRC handbook of combinatorial designs)

Theorem (Paduchikh (2009))

If $G = \mathrm{srg}(85, 14, 3, 2)$, ρ is an automorphism of G of prime order p, and Δ is the subgraph induced by the fixed points of ρ, then one of the following is true:

(1) *$p = 5$ or $p = 17$ and Δ is the empty graph;*

(2) *$p = 7$ and Δ is a 1–clique or $p = 5$ and Δ is a 5–clique;*

(3) *$p = 3$, Δ is a quadrangle or a 2×5 lattice, and in the last case the neighbourhoods of six vertices of Δ contain exactly two maximal cliques;*

(4) *$p = 2$.*

Orbit matrices $\quad srg(10, 3, 0, 1)$

$$\left[\begin{array}{c|ccc|ccc|ccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
\hline
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
\hline
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
\hline
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{array}\right]$$

Orbit matrices $\quad$ srg(10, 3, 0, 1)

$$\left[\begin{array}{c|ccc|ccc|ccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ \hline
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ \hline
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ \hline
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{array}\right]$$

$$C = [c_{ij}] = \left[\begin{array}{c|ccc}
0 & 0 & 0 & 1 \\ \hline
 & & & \\
 & & & \\
 & & &
\end{array}\right.$$

Orbit matrices $\quad \mathrm{srg}(10,3,0,1)$

$$\left[\begin{array}{c|ccc|ccc|ccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
\hline
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
\hline
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
\hline
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{array}\right]$$

$$C = [c_{ij}] = \left[\begin{array}{c|ccc}
0 & 0 & 0 & 1 \\
\hline
0 & 0 & 2 & 1 \\
 & & & \\
 & & &
\end{array}\right]$$

Orbit matrices $\quad \operatorname{srg}(10,3,0,1)$

$$\left[\begin{array}{c|ccc|ccc|ccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
\hline
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
\hline
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
\hline
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{array}\right]$$

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0 & 0 & 0 & 1 \\
\hline
0 & 0 & 2 & 1 \\
0 & 2 & 0 & 1 \\
 & & &
\end{array}\right.$$

Orbit matrices srg(10, 3, 0, 1)

$$\left[\begin{array}{c|ccc|ccc|ccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ \hline
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ \hline
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ \hline
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{array}\right]$$

$$C = [c_{ij}] = \left[\begin{array}{c|ccc}
0 & 0 & 0 & 1 \\ \hline
0 & 0 & 2 & 1 \\
0 & 2 & 0 & 1 \\
3 & 1 & 1 & 0
\end{array}\right],$$

Orbit matrices $\quad$ srg$(10,3,0,1)$

$$\left[\begin{array}{c|ccc|ccc|ccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
\hline
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
\hline
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
\hline
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{array}\right]$$

$$C = [c_{ij}] = \left[\begin{array}{c|ccc}
0 & 0 & 0 & 1 \\
\hline
0 & 0 & 2 & 1 \\
0 & 2 & 0 & 1 \\
3 & 1 & 1 & 0
\end{array}\right], \quad R = \left[\begin{array}{c|ccc}
0 & 0 & 0 & 3 \\
\hline
0 & 0 & 2 & 1 \\
0 & 2 & 0 & 1 \\
1 & 1 & 1 & 0
\end{array}\right],$$

Orbit matrices srg(10, 3, 0, 1)

$$\left[\begin{array}{c|ccc|ccc|ccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ \hline
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ \hline
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
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1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
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\end{array}\right]$$

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0 & 0 & 0 & 1 \\ \hline
0 & 0 & 2 & 1 \\
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3 & 1 & 1 & 0
\end{array}\right], \quad R = \left[\begin{array}{c|ccc}
0 & 0 & 0 & 3 \\ \hline
0 & 0 & 2 & 1 \\
0 & 2 & 0 & 1 \\
1 & 1 & 1 & 0
\end{array}\right], \quad N = \left[\begin{array}{c|ccc}
1 & 0 & 0 & 0 \\ \hline
0 & 3 & 0 & 0 \\
0 & 0 & 3 & 0 \\
0 & 0 & 0 & 3
\end{array}\right].$$

$$B^2 = (k-\mu)I + \mu J + (\lambda - \mu)B.$$

Lemma

$$CNC^T = S. \tag{1}$$

$$s_{ij} = \delta_{ij}(k-\mu)n_j + \mu n_i n_j + (\lambda-\mu)c_{ij}n_j. \tag{2}$$

$$s_{rr} = \sum_{k=1}^{t} c_{rk}^2 n_k, \tag{3}$$

$\mathrm{srg}(15, 6, 1, 3)$

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$p = 3$

fixed points $= 3$

$$C = \left[\begin{array}{ccc|cccc} 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ \hline 0 & 0 & 3 & 0 & 2 & 2 & 1 \\ 0 & 3 & 0 & 2 & 0 & 2 & 1 \\ 3 & 0 & 0 & 2 & 2 & 0 & 1 \\ 3 & 3 & 3 & 1 & 1 & 1 & 0 \end{array}\right]$$

Fixed prototype

$$\begin{cases} x_0 + x_1 = 3, \\ y_0 + y_1 = 4, \\ x_1 + 3y_1 = 6. \end{cases} \tag{4}$$

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Solutions:

$$(x_0, x_1, y_0, y_1) \in \{(0,3,3,1),(3,0,2,2)\}.$$

Fixed prototype

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Solutions:

$$(x_0, x_1, y_0, y_1) \in \{(0,3,3,1), (3,0,2,2)\}.$$

The first solution is not accepted since the diagonal of B is zero there has to be at least one zero in the fixed columns. Thus

$$x_0 \neq 0.$$

Non–fixed prototype

$$\begin{cases} x_0 + x_3 = 3, \\ y_0 + y_1 + y_2 + y_3 = 4, \\ x_3 + y_1 + 2y_2 + 3y_3 = 6, \\ 3x_3 + y_1 + 4y_2 + 9y_3 = s_{rr}/3. \end{cases} \tag{5}$$

Non–fixed prototype

$$\left\{\begin{array}{ccccccccccccc} x_0 & + & x_3 & & & & & & & & & = & 3, \\ & & & & y_0 & + & y_1 & + & y_2 & + & y_3 & = & 4, \\ & & x_3 & & & + & y_1 & + & 2y_2 & + & 3y_3 & = & 6, \\ & & 3x_3 & & & + & y_1 & + & 4y_2 & + & 9y_3 & = & s_{rr}/3. \end{array}\right. \tag{5}$$

$$s_{rr} = (k-\mu)p+\mu p^2+(\lambda-\mu)c_{rr}p.$$

$$s_{rr}/3 = 12 - 2c_{rr}$$

$$c_{rr} = 0, \text{ or } 2$$

Non–fixed prototype

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$$s_{rr} = (k-\mu)p + \mu p^2 + (\lambda-\mu)c_{rr}p.$$

$$s_{rr}/3 = 12 - 2c_{rr}$$

$$c_{rr} = 0, \text{ or } 2$$

$$(x_0, x_3, y_0, y_1, y_2, y_3) \in \{(0,3\ ,1,3,0,0), (1,2\ ,1,2,1,0), (2,1\ ,1,1,2,0), (3,0\ ,1,0,3,0), (3,0\ ,0,3,0,1)\}.$$

$$(x_0, x_1, y_0, y_1) \in \{(3, 0, 2, 2)\}.$$

$$CNC^T = S.$$

$$(x_0, x_3, y_0, y_1, y_2, y_3) \in$$
$$\{(0, 3\ , 1, 3, 0, 0),$$
$$(1, 2\ , 1, 2, 1, 0),$$
$$(2, 1\ , 1, 1, 2, 0),$$
$$(3, 0\ , 1, 0, 3, 0),$$
$$(3, 0\ , 0, 3, 0, 1)\}.$$

$$C = \left[\quad \right.$$

$$CNC^T = S.$$

$$(x_0, x_1, y_0, y_1) \in \{(3, 0, 2, 2)\}.$$

$$C = \left[\begin{array}{ccc|cccc} 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{array} \right]$$

$$\begin{aligned} &(x_0, x_3, y_0, y_1, y_2, y_3) \in \\ &\{(0, 3\ , 1, 3, 0, 0), \\ &\ (1, 2\ , 1, 2, 1, 0), \\ &\ (2, 1\ , 1, 1, 2, 0), \\ &\ (3, 0\ , 1, 0, 3, 0), \\ &\ (3, 0\ , 0, 3, 0, 1)\}. \end{aligned}$$

$$(x_0, x_1, y_0, y_1) \in \{(3,0,2,2)\}.$$

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$$(3,0\ ,1,0,3,0),$$
$$(3,0\ ,0,3,0,1)\}.$$

$$C = \left[\begin{array}{ccc|cccc} 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ & & & & & & \\ & & & & & & \end{array}\right.$$

$$(x_0, x_1, y_0, y_1) \in \{(3, 0, 2, 2)\}.$$

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$$(x_0, x_3, y_0, y_1, y_2, y_3) \in$$
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$$(2, 1\ , 1, 1, 2, 0),$$
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$$(3, 0\ , 0, 3, 0, 1)\}.$$

$$C = \left[\begin{array}{ccc|cccc} 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ \hline & & & & & & \\ & & & & & & \\ & & & & & & \end{array}\right.$$

$$(x_0, x_1, y_0, y_1) \in \{(3,0,2,2)\}.$$

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$$(3,0\ ,0,3,0,1)\}.$$

$$CNC^T = S.$$

$$C = \left[\begin{array}{ccc|cccc} 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ \hline 0 & 0 & 3 & 0 & 2 & 2 & 1 \\ & & & & & & \\ & & & & & & \end{array}\right.$$

$$(x_0, x_1, y_0, y_1) \in \{(3, 0, 2, 2)\}.$$

$$CNC^T = S.$$

$$(x_0, x_3, y_0, y_1, y_2, y_3) \in$$
$$\{(0, 3\ , 1, 3, 0, 0),$$
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$$(2, 1\ , 1, 1, 2, 0),$$
$$(3, 0\ , 1, 0, 3, 0),$$
$$(3, 0\ , 0, 3, 0, 1)\}.$$

$$C = \left[\begin{array}{ccc|cccc} 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ \hline 0 & 0 & 3 & 0 & 2 & 2 & 1 \\ 0 & 3 & 0 & 2 & 0 & 2 & 1 \\ 3 & 0 & 0 & 2 & 2 & 0 & 1 \\ 3 & 3 & 3 & 1 & 1 & 1 & 0 \end{array}\right]$$

Example

$\mathrm{srg}(15, 6, 1, 3)$

$p = 5$

$\phi = 0$

$$C = \begin{pmatrix} 0 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{pmatrix}$$

Example

srg(15, 6, 1, 3)

$p = 5$

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$$C = \begin{pmatrix} 0 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{pmatrix}$$

$$B = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & X & X & X & X & X & X & X & X & X & X \\
0 & 0 & 0 & 0 & 0 & X & X & X & X & X & X & X & X & X & X \\
0 & 0 & 0 & 0 & 0 & X & X & X & X & X & X & X & X & X & X \\
0 & 0 & 0 & 0 & 0 & X & X & X & X & X & X & X & X & X & X \\
0 & 0 & 0 & 0 & 0 & X & X & X & X & X & X & X & X & X & X \\
X & X & X & X & X & 0 & 1 & 0 & 0 & 1 & X & X & X & X & X \\
X & X & X & X & X & 1 & 0 & 1 & 0 & 0 & X & X & X & X & X \\
X & X & X & X & X & 0 & 1 & 0 & 1 & 0 & X & X & X & X & X \\
X & X & X & X & X & 0 & 0 & 1 & 0 & 1 & X & X & X & X & X \\
X & X & X & X & X & 1 & 0 & 0 & 1 & 0 & X & X & X & X & X \\
X & X & X & X & X & X & X & X & X & X & X & X & X & X & X \\
X & X & X & X & X & X & X & X & X & X & X & X & X & X & X \\
X & X & X & X & X & X & X & X & X & X & X & X & X & X & X \\
X & X & X & X & X & X & X & X & X & X & X & X & X & X & X \\
X & X & X & X & X & X & X & X & X & X & X & X & X & X & X
\end{pmatrix}.$$

Pruning the backtrack search

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- $B^2 = (k - \mu)I + \mu J + (\lambda - \mu)B.$
- Isomorph rejection.
- Positive semidefinite test.

Correction test

Aut. group size	Number of SRGs McKay program	Number of SRGs the SRG program
1	28	Not Applicable
2	37	37
3	14	14
4	51	51
8	16	16
12	5	5
16	5	5
21	2	2
24	9	9
32	1	1
36	1	1
48	5	5
64	1	1
72	1	1
144	1	1
216	1	1
432	1	1
12096	1	1

Automorphis group statistics of all SRG(36, 14, 4, 6)

p	#fix point	#orb matrix	#srg found
3	1	0	
	4	2	0
	⋮	⋮	
	28	0	
5	0	3	0
	5	1	0
	10	0	
	⋮	⋮	
	30	0	
7	1	8	0
	8	0	
	15	0	
	22	0	
	29	0	
11	8	0	
	19	0	
	30	0	
13	7	0	
	20	0	
	33	0	
17	0	2	0

Table: Results on the automorphisms of $\mathrm{srg}(85, 14, 3, 2)$.

Theorem (Paduchikh (2009))

If $G = \mathrm{srg}(85, 14, 3, 2)$, ρ is an automorphism of G of prime order p, and Δ is the subgraph induced by the fixed points of ρ, then one of the following is true:

(1) *$p = 5$ or $p = 17$ and Δ is the empty graph;*

(2) *$p = 7$ and Δ is a 1–clique or $p = 5$ and Δ is a 5–clique;*

(3) *$p = 3$, Δ is a quadrangle or a 2×5 lattice, and in the last case the neighbourhoods of six vertices of Δ contain exactly two maximal cliques;*

(4) *$p = 2$.*

From out work, $p = 2$ is the only possible prime divisor of $|\mathrm{Aut}(G)|$.

G	possible primes $\{p : p\mid \lvert Aut(G)\rvert\}$
$\mathrm{srg}(65, 32, 15, 16)$	2,3,5
$\mathrm{srg}(69, 20, 7, 5)$	2,3
$\mathrm{srg}(75, 32, 10, 16)$	2,3
$\mathrm{srg}(76, 30, 8, 14)$	2,3
$\mathrm{srg}(76, 35, 18, 14)$	2,3,5
$\mathrm{srg}(85, 14, 3, 2)$	2
$\mathrm{srg}(85, 30, 11, 10)$	2,3,5,17
$\mathrm{srg}(85, 42, 20, 21)$	2,3,5,7
$\mathrm{srg}(88, 27, 6, 9)$	2,3,5,11
$\mathrm{srg}(95, 40, 12, 20)$	2,3,5
$\mathrm{srg}(96, 35, 10, 14)$	2,3,5
$\mathrm{srg}(96, 38, 10, 18)$	2,3,5
$\mathrm{srg}(96, 45, 24, 18)$	2,3,5
$\mathrm{srg}(99, 14, 1, 2)$	2,3
$\mathrm{srg}(99, 42, 21, 15)$	2,3,5,7,11
$\mathrm{srg}(100, 33, 8, 12)$	2,3,5,11

Table: Summary of results for unknown strongly regular graphs

Some New $\mathrm{srg}(49, 18, 7, 6)$

Aut. group size	Number of SRGs	New?	Aut. Group
10	1	no	D_{10}
15	3	2 new	C_{15}
21	1	yes	$7:3$
30	1	yes	$D_{10} \times C_3$
63	1	yes	$7:3 \times C_3$
126	1	yes	
1008	1	no	
1764	1	no	

Table: Automorphism group size statistics of all $\mathrm{srg}(49, 18, 7, 6)$ with automorphism group size divisible by 5 and 7 obtained from the SRG program.