Totally Silver Graphs

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This is a joint work with:

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Silver matrix:

1	2	3	4	5	6
7	1	5	3	6	4
8	10	1	6	4	2
9	8	11	1	2	5
10	11	9	7	1	3
11	9	7	10	8	1

Silver matrix:

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An IMO Problem

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- There is no silver matrix of order 1997.
- There exist silver matrices of infinitely many different orders.

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Theorem (Mahdian, Mahmoodian; 2000)

A silver matrix of order n exists, if and only if $n = 1$ or $2|n$.

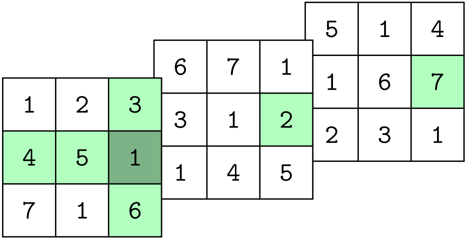
Silver Cubes

1	2	3
4	5	1
7	1	6

6	7	1
3	1	2
1	4	5

5	1	4
1	6	7
2	3	1

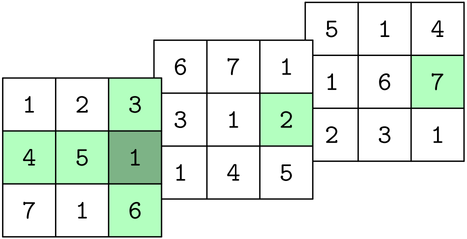
Silver Cubes



Theorem (G, Goddyn, Mahmoodian, Verdian; 2008)

A silver cube of order n exists, if $n = 2^a 3^b 5^c$.

Silver Cubes



Theorem (G, Goddyn, Mahmoodian, Verdian; 2008)

A silver cube of order n exists, if $n = 2^a 3^b 5^c$.

Theorem (Ventullo, Khodkar; 2007)

A silver cube of order 7 exists.

Silver Cubes

Definition

An *(n, d)–silver cube* is any

$$c : V(K_n^d) \to \{0, 1, \ldots, d(n-1)\}$$

in which every vertex of a *diagonal* is *rainbow*.

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Open Problems

- K_{11}^3 (K_p^3 is silver where $p \geqslant 11$ is a prime)

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Open Problems

- K_{11}^3 (K_p^3 is silver where $p \geqslant 11$ is a prime)
- Q_{20} (Q_n is not silver where $4|n$ and $n \neq 2^t$)

Silver Graphs

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Given an r–regular G and a diagonal I,

$$c : V(G) \to \{0, 1, \ldots, r\}$$

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is a *silver colouring* if every $v \in I$ is rainbow. *Totally silver* if every $v \in V(G)$ is rainbow.

G is said to be *(totally) silver*, if it admits a (totally) silver colouring.

Observation

G is totally silver iff G is *domatically full*, i.e. $V(G)$ admits $\delta(G) + 1$ disjoint dominating sets.

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G is totally silver iff $\chi(G^2) = r+1$ where G^2 is the *square* of G.

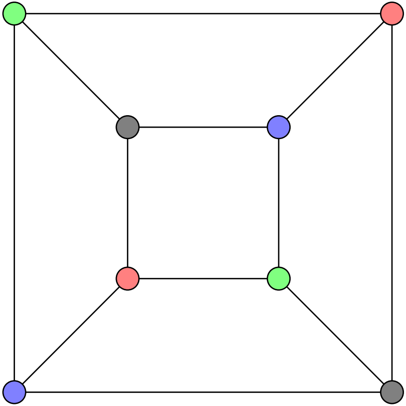
Examples of Totally Silver Graphs

1	2	3
4	5	1
7	1	6

6	7	1
3	1	2
1	4	5

5	1	4
1	6	7
2	3	1

Examples of Totally Silver Graphs



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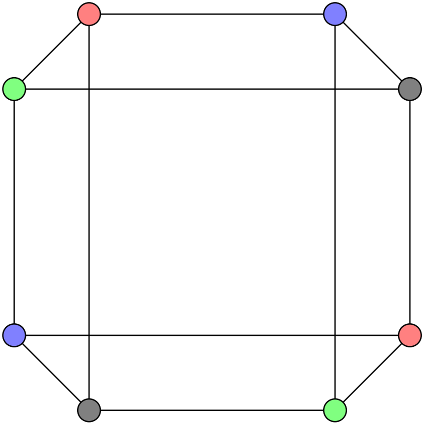

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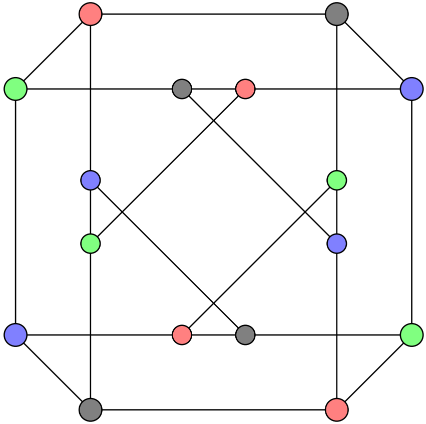

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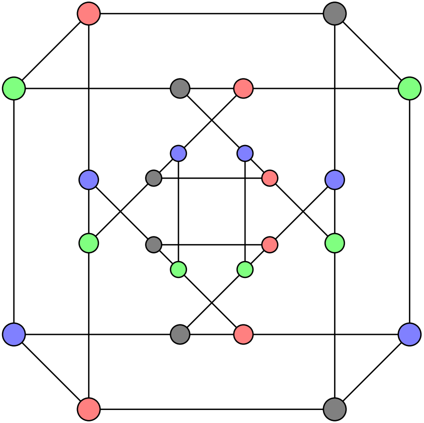

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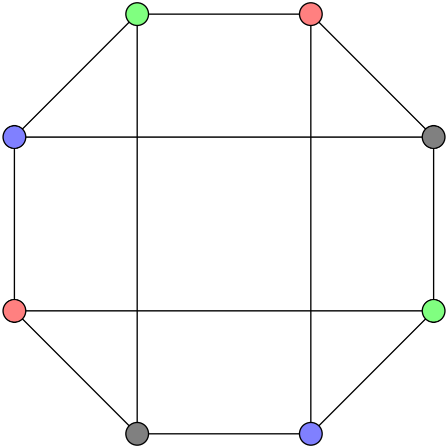

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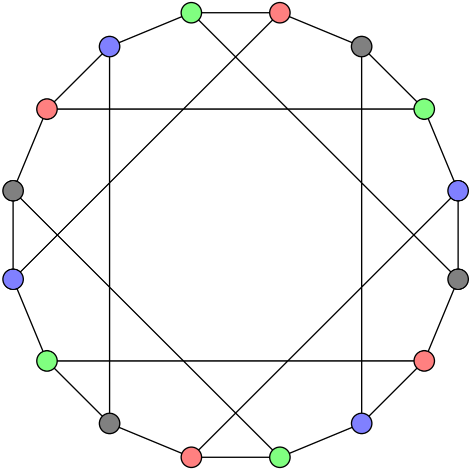

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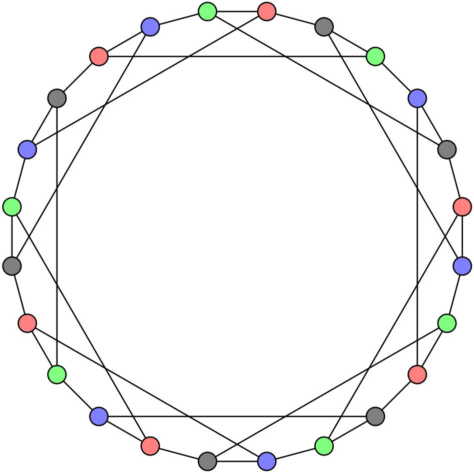
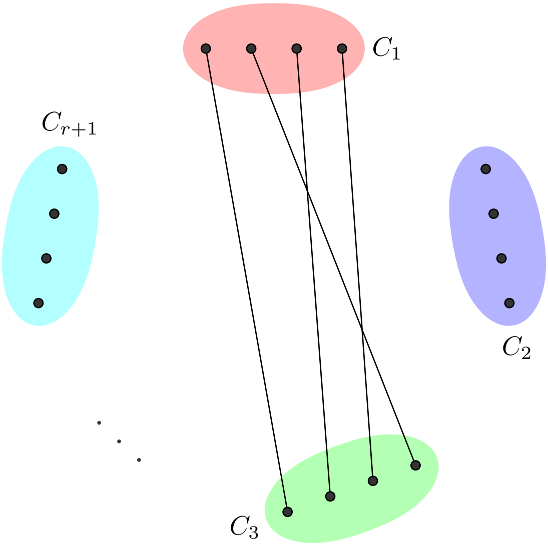


Characterization of Totally Silver Graphs



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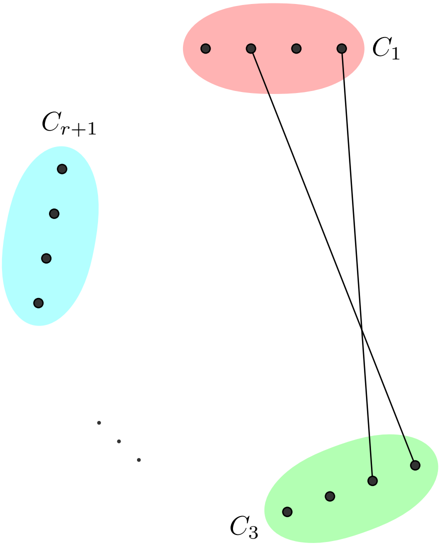

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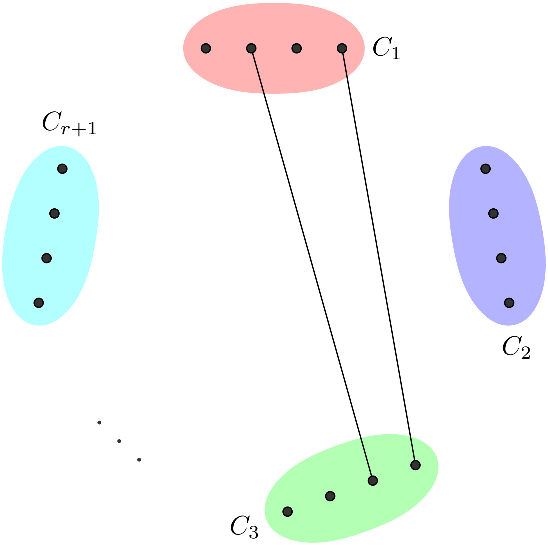

Characterization of Totally Silver Graphs

Theorem

Every r–regular totally silver graph can be obtained by a sequence of (coloured) 2–switches from a disjoint union of copies of $r+1$–cliques.

- 3– and 4–cycles can be reduced.

Totally Silver Cubic Graphs

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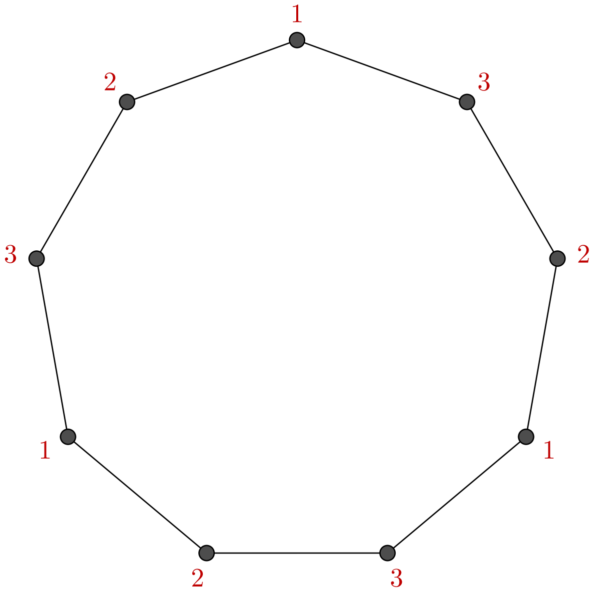
So a *nontrivial* totally silver cubic graph is bridgeless with girth at least 6.

Question

Do there exist totally silver cubic graphs of high girth?

An Example with Girth 9



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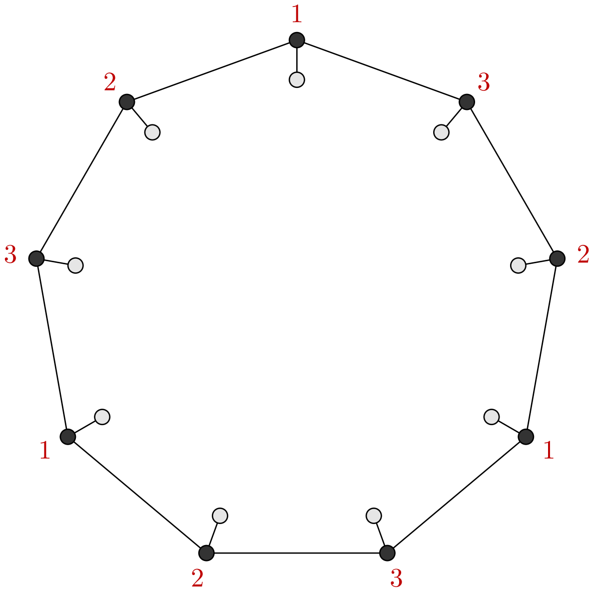

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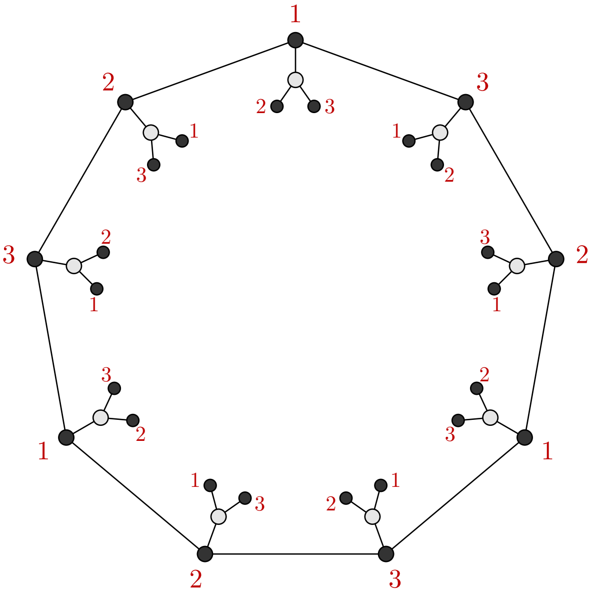

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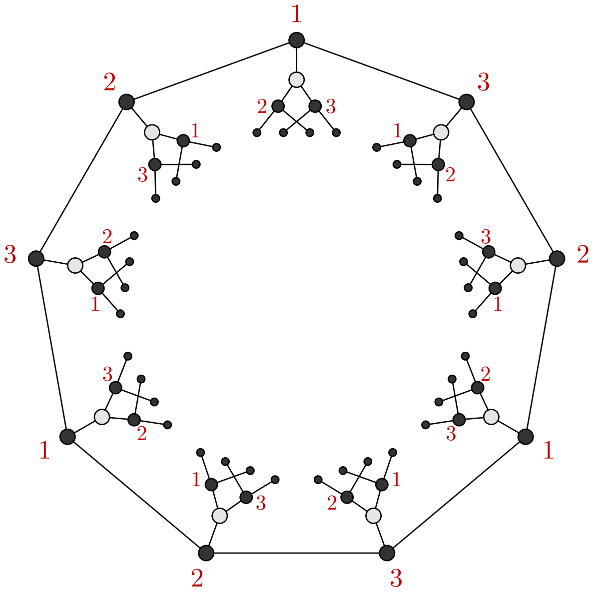

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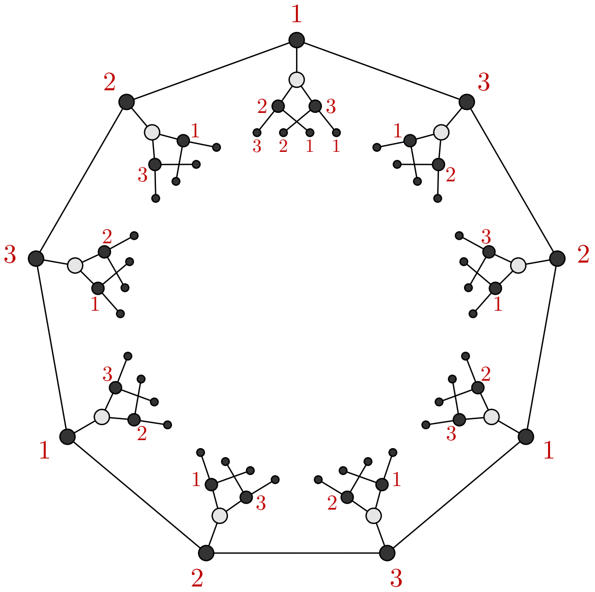

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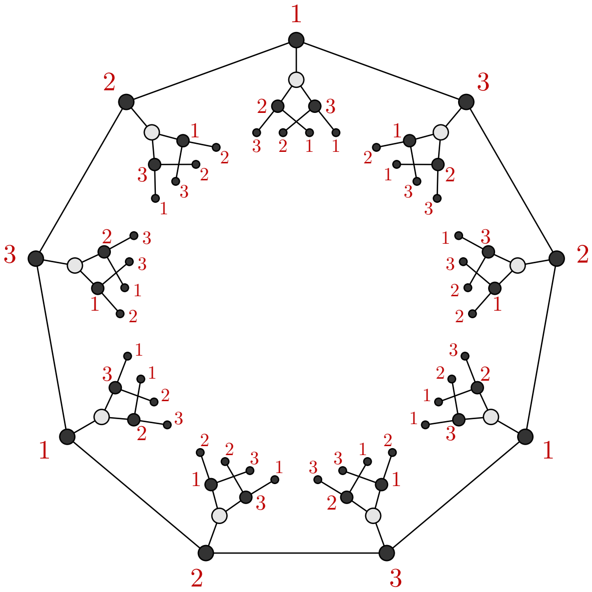

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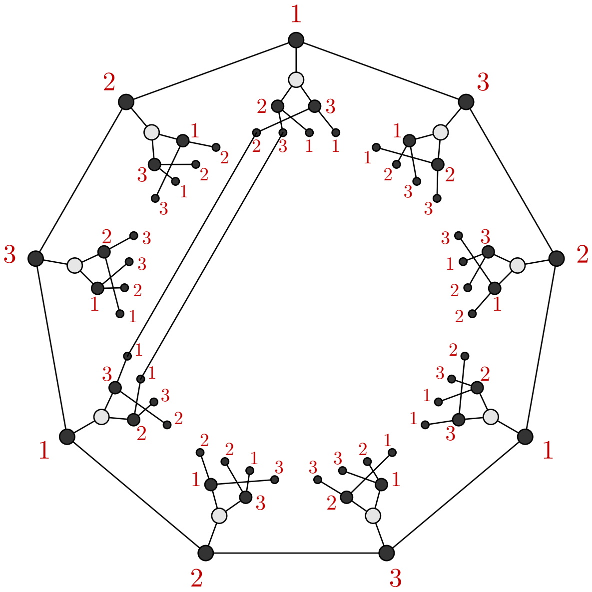

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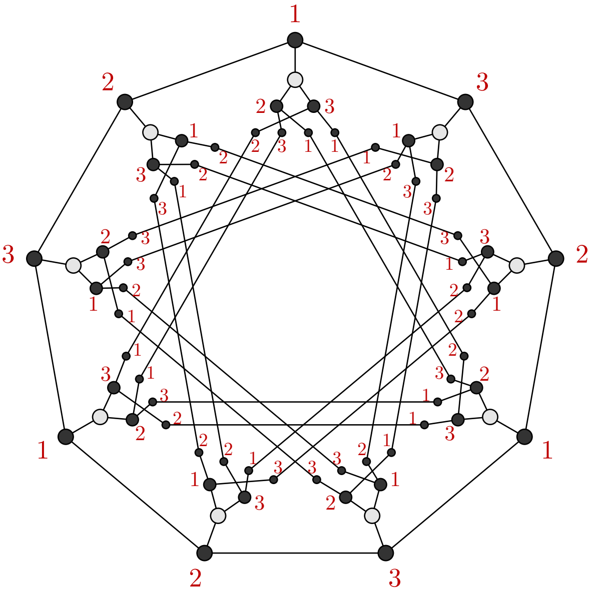

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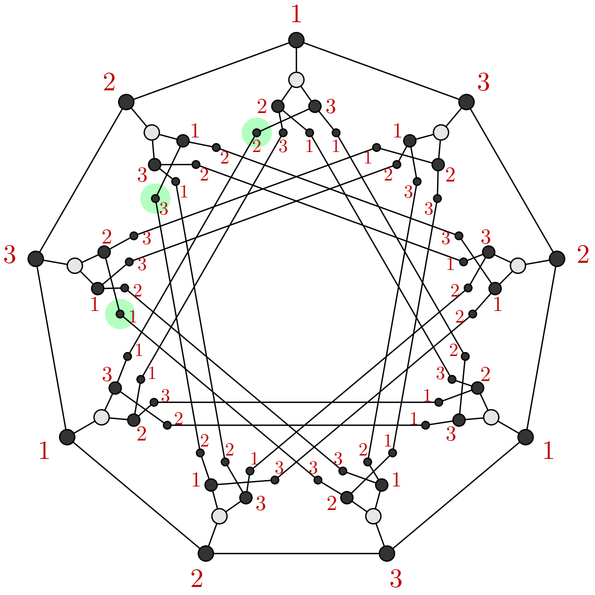

Thank you!