ON MINIMUM AND MAXIMUM VALUES OF γ-LABELINGS OF GRAPHS

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CANAdian Discrete and Algorithmic Mathematics Conference, May 25-28, 2009

OUTLINE

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γ-LABELINGS

A γ-labeling of a graph G with m edges is a one-to-one function from $V(G)$ to $\{0, 1, \ldots, m\}$.

A γ-labeling exists for a graph if the order is at most one more than the size. Any connected graph has a γ-labeling.

A γ-labeling induces a labeling of the edges. The edge uv is labeled with $|label(u) - label(v)|$.

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MAXIMUM AND MINIMUM LABELINGS

The value of a γ-labeling is the sum of the induced labels of the edges.

$val_{max}(G)$ = maximum value over all γ-labelings of G
$val_{min}(G)$ = minimum value over all γ-labelings of G

The spectrum of G is the set of all possible values of γ-labelings of G.

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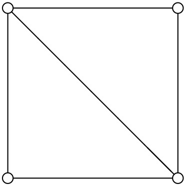
BANDWIDTH

I recently learned that this is very similar to the bandwidth of a labeling.

If the vertices of G are labeled with $\{1, 2, \ldots, |V(G)|\}$, the bandwidth is the sum of the induced edge labels.

Example $Val_{min}(G)$

A γ-labeling will use labels from $\{0, 1, 2, 3, 4, 5\}$.



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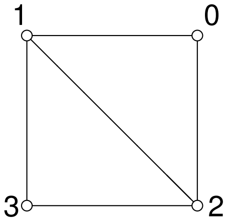

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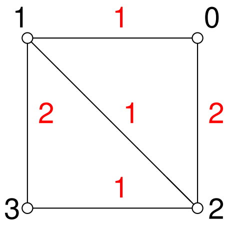
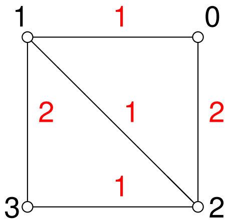


EXAMPLE $Val_{min}(G)$

A γ-labeling will use labels from $\{0, 1, 2, 3, 4, 5\}$.
$val_{min}(G) = 7$.



EXAMPLE $Val_{max}(G)$

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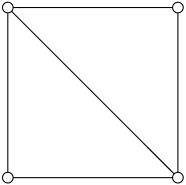

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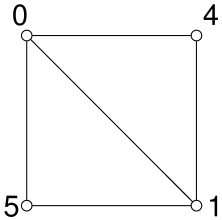

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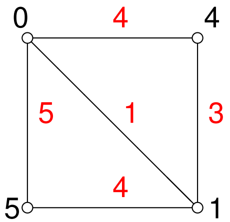

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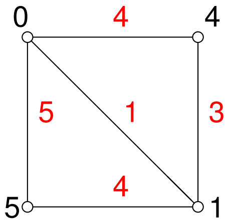
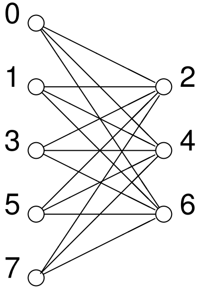
$val_{max}(G) = 17$.



$val_{min}(K_{n_1,n_2})$

For $n_1 \geq n_2$,

$$val_{min}(K_{n_1,n_2}) = \frac{n_2(2n_2^2+1)}{3} + (n_1 - n_2)n_2^2 + \left\lfloor \frac{(n_1-n_2)^2}{4} \right\rfloor n_2$$



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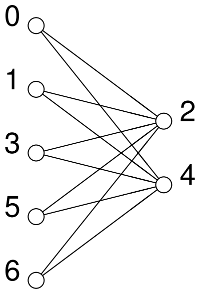
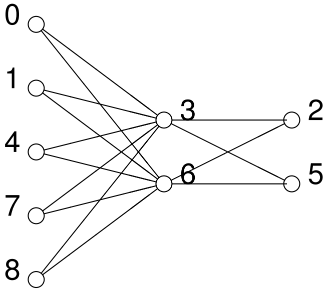


$val_{min}(K_{n_1,n_2,\ldots,n_r})$

A similar labeling produces the minimum value for a complete multipartite graph. (Some edges left out for clarity)



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Sketch of proof: WLOG, labels are consecutive and start at 0. Given any labeling with these labels, it can be converted into this labeling by a series of swaps which does not increase the value.

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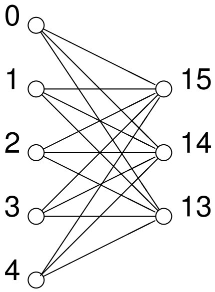
$val_{max}(K_{n_1,n_2})$

$$val_{max}(K_{n_1,n_2}) = n_1 n_2(n_1 n_2 - \frac{1}{2}n_1 - \frac{1}{2}n_2 + 1)$$



Partial Results on Spectra

For every even positive integer n, every number in the spectrum of $K_{n,n}$ is even.

Sketch of proof: First, we show that any γ-labeling of $K_{n,n}$ can be converted into any other γ-labeling by a series of moves, where each move is one of the following:

- add or subtract 1 from a single label
- swap two consecutive labels used in opposite partite sets

Then we show that each of these moves changes the value by an even number.

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Partial Results on Spectra

For every even positive integer n, every even integer from $\frac{n(2n^2+1)}{3}$ to $\frac{n^2(2n^2-n+2)}{4}$ is in the spectrum of $K_{n,n}$.

From previous results, $val_{min}(K_{n,n}) = \frac{n(2n^2+1)}{3}$ and $val_{max}(K_{n,n}) = n^2(n^2 - n + 1)$.

Partial Results on Spectra

For any odd integer $n \geq 3$, every integer from $\frac{n(n+1)(3n-1)}{4}$ to $\frac{n(2n^3+n^2-2n+3)}{4}$ is in the spectrum of $K_{n,n}$.

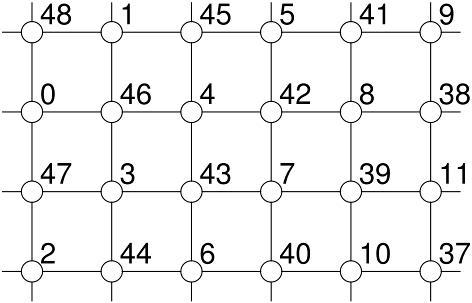
From previous results, $val_{min}(K_{n,n}) = \frac{n(2n^2+1)}{3}$ and $val_{max}(K_{n,n}) = n^2(n^2 - n + 1)$.

Partial Results on Spectra

For any integer $n \geq 2$, the value $\frac{n(2n^2+1)}{3} + 1$ is not in the spectrum of $K_{n,n}$. Furthermore, the values $n^2(n^2 - n + 1) - i$, for $1 \leq i \leq n - 1$, are not in the spectrum of $K_{n,n}$.

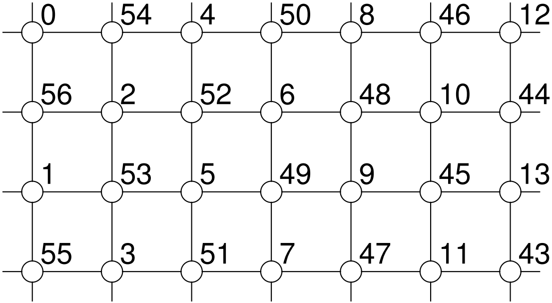
$Val_{max}(C_a \times C_b)$

If a and b are even integers, $a \geq 4$ and $b \geq 4$, then $val_{max}(C_a \times C_b) = ab(3ab + 2)$.



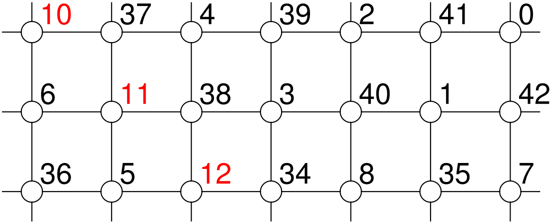
$Val_{max}(C_a \times C_b)$

If $a \geq 3$ is odd and $b \geq 4$ is even, then
$val_{max}(C_a \times C_b) = 3a^2b^2 - ab^2 + 2ab - \frac{1}{2}b^2 - b.$



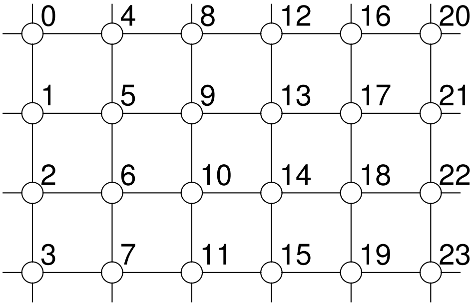
$Val_{max}(C_a \times C_b)$

If $a \geq 3$ and $b \geq 3$ are odd integers, then $val_{max}(C_a \times C_b) = 3a^2b^2 - ab^2 - a^2b + 2ab - a - b - \frac{1}{2}(a^2 + b^2)$.



$Val_{min}(C_a \times C_b)$

If $a \geq b \geq 3$ are integers, then
$val_{min}(C_a \times C_b) = 2a(b-1) + 2b^2(a-1)$.



A sampling of other results on $\gamma-$labeling

G. Chartrand, D. Erwin, D.W. VanderJagt, and P. Zhang found the spectrum of stars $K_{1,n}$ and the maximum and minimum value of paths, cycles, and complete graphs.

C.M. de Fonseca, Varaporn Saenpholphat and Ping Zhang determined the spectra of paths, cycles, and complete graphs.

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Futaba Okamoto, Ping Zhang, and Varaporn Saenpholphat defined balanced $\gamma-$labeling of digraphs and characterized which digraphs have balanced $\gamma-$labelings.

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